May 31, 2018

Continuous Growth and natural logarithms

Objective - Use e and the natural logarithm to model continuous growth.

WHAT IF IT'S COMPOUNDED CONTINUOUSLY?!

that means it's compounded constantly, every second of everyday!

Lets see what happens if we invest \$1, with an interest rate of 100% over the course of one year....

How often compounded	Computation $\left(A=P\left(1+\frac{r}{n}\right)^{nt}\right)$ $\left(A=1\left(1+\frac{1}{n}\right)^{n(1)}\right)$
Yearly (n=1)	$A=1\left(1+\frac{1}{1}\right)^1=1(2)=2$
Biannually (n=2)	$A=1\left(1+\frac{1}{2}\right)^2=1(1.5)^2=2.25$
Quarterly (n=4)	$A=1\left(1+\frac{1}{4}\right)^4=1(1.25)^4=2.441$
Monthly (n=12)	$A=1\left(1+\frac{1}{12}\right)^{12}=1(1.08333)^{12}=2.613$
Daily (n=365)	$A=1\left(1+\frac{1}{365}\right)^{365}=1(1.0027)^{365}=2.715$
Hourly (n=8760)	$A=1\left(1+\frac{1}{8760}\right)^{8760}=1(1.000114)^{8760}=2.718$
every minute (n=525600)	$A=1\left(1+\frac{1}{525600}\right)^{525600}=2.7183...$

*As you can see the more often we compound interest, the closer this number gets to...

2.718282247254

(we call this e (similar to the idea of π))

- its so important it even has its own logarithm!!!

$$\log_e x = \ln x$$

* ln stands for natural logarithm

ex) $\log_e 5 = \ln(5)$ ⟵ lets have $x=5$

$\frac{\log(5)}{\log(e)} = \ln(5)$ ⟵ find ln right below the log button and find e by pressing 2nd, ln

$1.609 = 1.609$ ✓

($e = e^1$)

* we use continuous growth or decay to represent....

1) interest
2) population
3) rate of dissolving
4) cooling and heating
5) radioactive decay

continuous growth/decay formula:

$$A = Pe^{rt}$$

t ⟵ time (years)

r — growth/decay rate (decimal)

A — amount in account

P — start value/ amount deposited

example) I put $1500 into a savings account that gets 4% interest that compounds continuously

$P=$; $r = 4\% = 0.04$

$A = Pe^{rt}$ formula!

- Write a function:

$$A = Pe^{rt}$$
$$= 1500e^{0.04t}$$

- How much will be in the account after 10 years? ($t=10$)

$$A = 1500e^{0.04(10)}$$
$$= 1500e^{0.40}$$
$$= \$2237.74$$

can find e by pressing 2nd, Ln on graphing calculator!

- How long will it take for the account to double? ($A = 3000$)

$$A = 1500e^{0.04t}$$
$$\frac{3000}{1500} = \frac{1500e^{0.04t}}{1500}$$
$$2 = e^{0.04t}$$
$$\boxed{e^{0.04t} = 2}$$

rewrite to turn into logarithm!

$$\log_e 2 = 0.04t$$

$\log_e x = \ln x$

$$\ln(2) = 0.04t$$
$$\frac{.693}{0.04} = \frac{0.04t}{0.04}$$

$17.33 = t$ ← It will take 17.33 years for your money to double!.

example) On July, 1999 the world population reached

P=6 6 billion people and was growing exponentially.

The rate of growth was 1.21%

$r = 1.21\% = 0.0121$

population = continuously compounded

$A = Pe^{rt}$

- Write a function:

$$A = Pe^{rt}$$
$$= 6e^{0.0121t}$$

- Based on this model, what was the world population on July 18, 2008?

(how much time has passed from 1999 to 2008?

$t = 2008 - 1999 = 9$

$$A = 6e^{0.0121(9)}$$
$$= 6e^{0.1089}$$
$$= 6.69$$

← around 6.69 billion people!

- When will the world population reach 7 billion?

$(A = 7)$

$$\frac{7}{6} = \frac{6e^{0.0121t}}{6}$$

$$\frac{7}{6} = e^{0.0121t}$$

rewrite to turn into logarithm

$$e^{0.0121t} = \frac{7}{6}$$

$$\log_e \frac{7}{6} = 0.0121t$$ ← $\log_e x = \ln x$

$$\ln\left(\frac{7}{6}\right) = 0.0121t$$

$$\frac{0.15415}{0.0121} = \frac{0.0121t}{0.0121}$$

$$12.74 = t$$ ← it will take 12.74 years past 1999 to reach 7 billion!

example) If 600g (P) of a radioactive substance (continuously) are present initially and 3 years (t) later only 300 g (A = amount left) remain, how much of the substance will be present after 6 years? How about 10 years?

function: $A = Pe^{rt}$ ← dealing with radioactive substance

$$\frac{300}{600} = \frac{600e^{r(3)}}{600}$$ ← solving for the decay rate!

$$\frac{1}{2} = e^{r(3)}$$

$$\boxed{e^{3r} = \frac{1}{2}}$$

$$\log_e \frac{1}{2} = 3r$$

$$\ln\left(\frac{1}{2}\right) = 3r$$

$$\frac{-0.693}{3} = \frac{3r}{3}$$

$$-0.23 = r$$ ← negative because it is decaying!

$$\boxed{A = 600e^{-0.23t}}$$

- After 6 years? $(t=6)$

$A = 600e^{-0.23(6)}$

$= 600e^{-1.38}$

$= 150.947g \leftarrow$ amount left over!

- After 10 years? $(t=10)$

$A = 600e^{-0.23(10)}$

$= 600e^{-2.3}$

$= 60.155 \leftarrow$ amount left over!