Exponential + logarithmic functions unit practice test answers

1) $32^{\frac{-4}{5}} = \frac{1}{32^{4/5}} = \frac{1}{(32^4)^{1/5}} = \frac{1}{(1048576)^{1/5}} = \frac{1}{\sqrt[5]{1048576}} = \frac{1}{16}$

2) $\sqrt[4]{2x^6y^8} = (2x^6y^8)^{1/4} = (2)^{1/4}(x^6)^{1/4}(y^8)^{1/4} = 2^{1/4}x^{3/2}y^2$

factor 27 and 81

3) $\sqrt[4]{27} \cdot \sqrt[3]{81} = \sqrt[4]{3^3} \cdot \sqrt[3]{3^4} = 3^{3/4} \cdot 3^{4/3} = 3^{3/4+4/3} = 3^{\frac{25}{12}}$

$= \sqrt[12]{3^{25}}$

4) $3^{3\pi-1} \cdot 3^{\pi+4} = 3^{(3\pi-1+\pi+4)}$

$= 3^{4\pi+3}$

5) $x^{2/5} + 5 = 14$

$-5 \quad -5$

$x^{2/5} = 9$

$(\sqrt[5]{x^2})^5 = (9)^5$

$\sqrt{x^2} = \sqrt{59049}$

$\boxed{x = \pm 243}$

6) $36^x = \frac{1}{\sqrt{6}}$

$36^x = \frac{1}{6^{1/2}}$

$36^x = 6^{-1/2}$

$(6^2)^x = 6^{-1/2}$

Ignore bases, set exponents equal to eachother

$6^{2x} = 6^{-1/2}$ ← if $a^{f(x)} = a^{g(x)}$, then $f(x) = g(x)$

$\frac{2x}{2} = \frac{-1/2}{2}$

$\boxed{x = -1/4}$

Inverse operations

$+ \longleftrightarrow -$

$\times \longleftrightarrow \div$

$\sqrt{x} = \sqrt[2]{x} \longleftrightarrow x^2$

$\sqrt[3]{x} \longleftrightarrow x^3$

⋮

Converting roots to exponents

$\sqrt{x} = \sqrt[2]{x} = x^{1/2}$

$\sqrt[3]{x} = x^{1/3}$

$\sqrt[4]{x} = x^{1/4}$

⋮

7) $f(x) = a(b)^x$ ← # of repeated products

initial value ↗ growth factor

* If $b > 1$ - exponential growth
* If $b < 1$ - exponential decay

a = initial value = 10,000
b = growth factor = decreasing 20% per year $1 - .20 = .80$
= increasing 80% per year ← .80

x = 2 years

$f(2) = 10{,}000(0.8)^2 = \boxed{6{,}400 \text{ mice}}$

8) $f(x) = 10{,}000\underbrace{(0.8)^x}_{\text{"}\frac{1}{2} = 0.5}$

"half the size" = multiplying by ½

OR $\frac{5{,}000}{10{,}000} = \frac{10{,}000(0.8)^x}{10{,}000}$

$\frac{1}{2} = (0.8)^x$

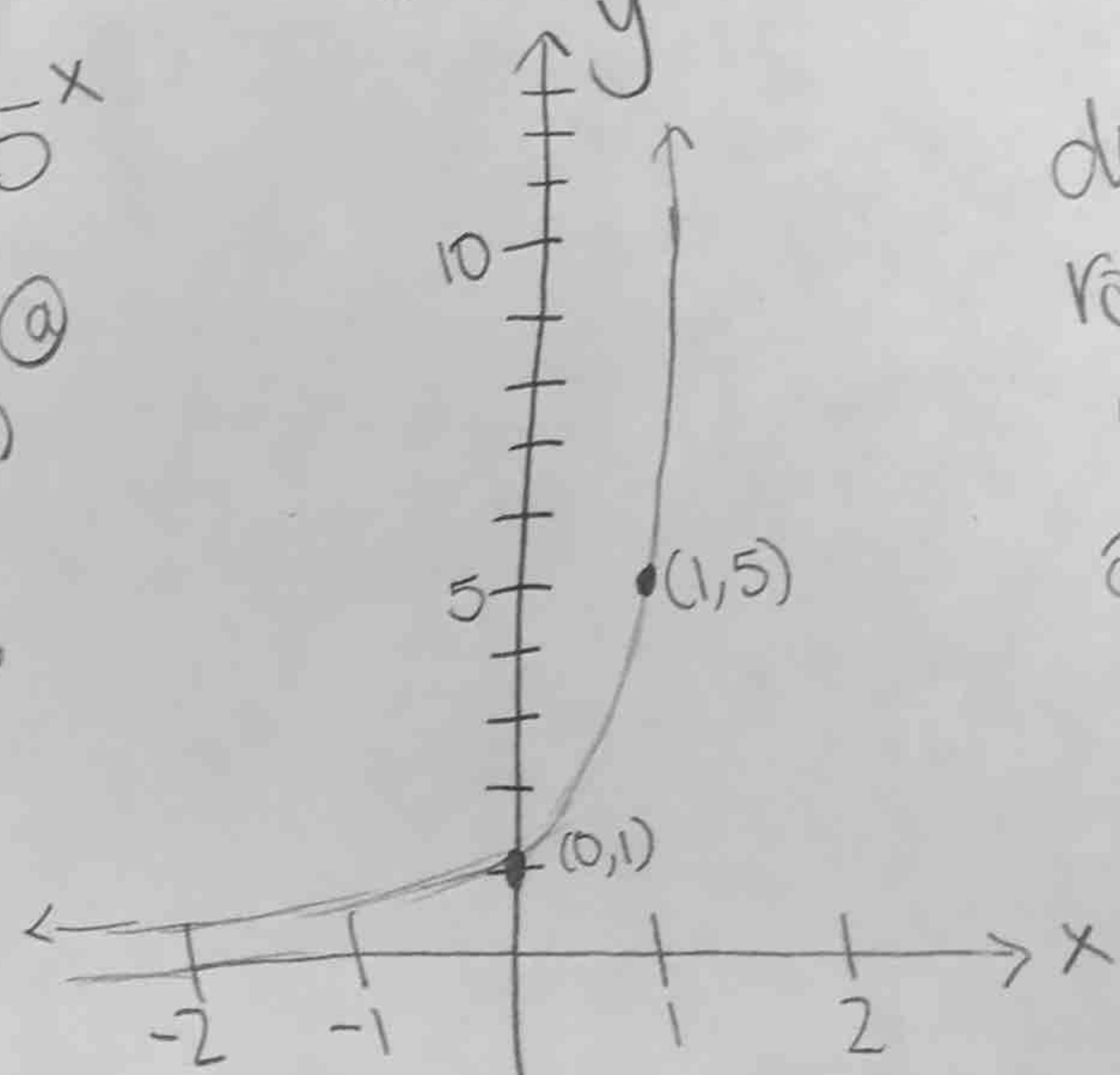
$0.8^x = 0.5 \Rightarrow \log_{0.8} 0.5 = x \Rightarrow \frac{\log 0.5}{\log 0.8} = x \Rightarrow x = 3.1$

After 3.1 years, the population will be half the original size

9) $f(x) = 5^x$

start @ (0,1)

$x = 1 \rightarrow$
$f(1) = 5^1 = 5$
(1,5)



domain - $(-\infty, \infty)$
range - $(0, \infty)$
y-intercept - $y = 1$
asymptote - $y = 0$

* Must include starting point and one other point!!

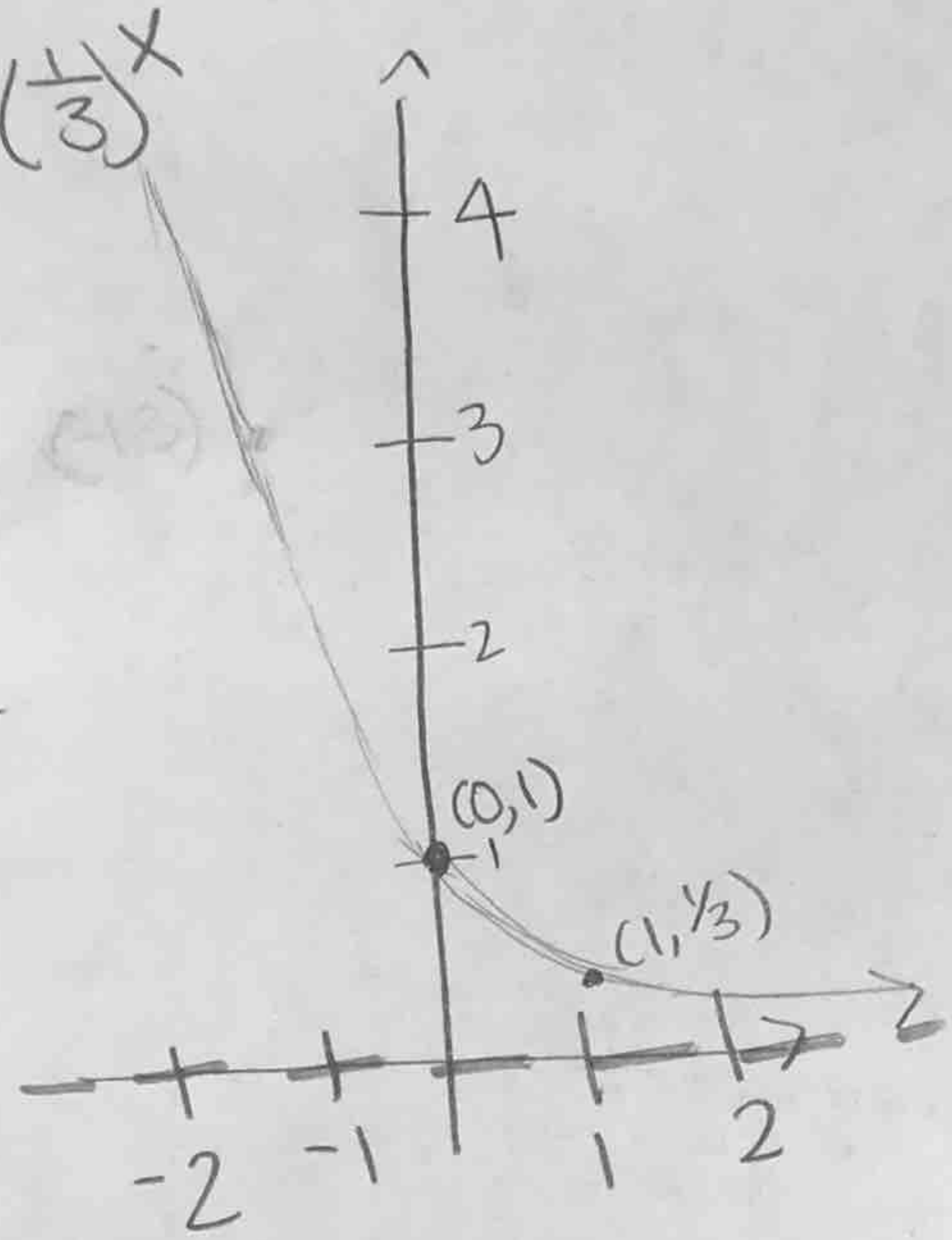
10) $f(x) = \left(\frac{1}{3}\right)^x$

start @ (0,1)

$x = 1 \rightarrow$

$f(1) = \left(\frac{1}{3}\right)^1 = \frac{1}{3}$

$\left(1, \frac{1}{3}\right)$



domain - $(-\infty, \infty)$

range - $(0, \infty)$

y-intercept - $y = 1$

asymptote - $y = 0$

11) $f(x) = -\left(\frac{1}{3}\right)^{x-4} + 7$

flipped over x-axis

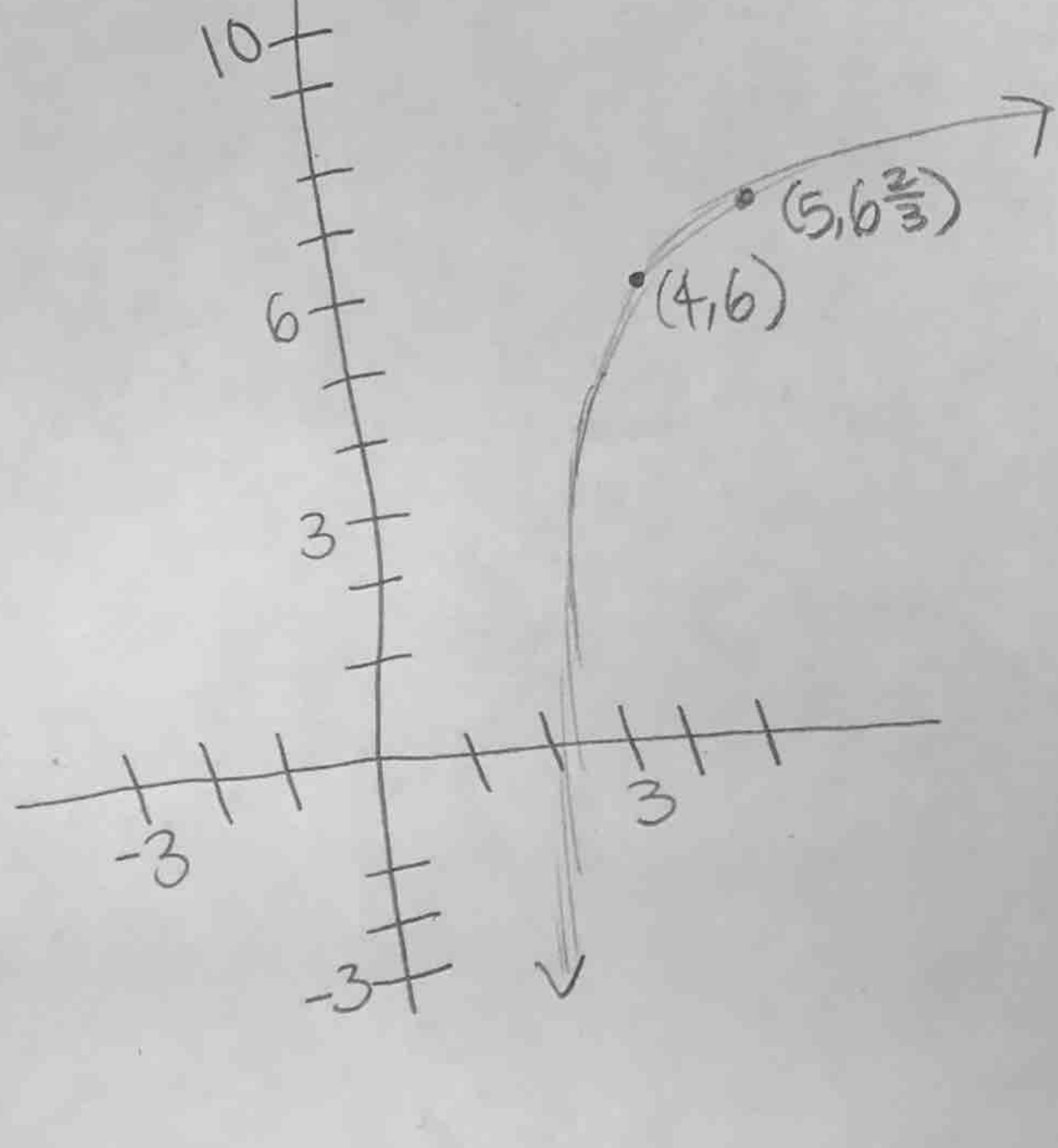
right 4

up 7

given the parent function

	flipped over x-axis $(x, -y)$	right 4 $(x+4, y)$	up 7 $(x, y+7)$
Initial points - $(0,1) \longrightarrow$	$(0,-1) \longrightarrow$	$(4,-1) \longrightarrow$	$(4,6)$
$(1,\frac{1}{3}) \longrightarrow$	$(1,-\frac{1}{3}) \longrightarrow$	$(5,-\frac{1}{3}) \longrightarrow$	$(5,6\frac{2}{3})$

New translated points!



12) $f(x) = b^x$

* if $b>1$ - exponential growth

  - as b gets larger, the graph increases more rapidly (steeper)

* If $0<b<1$ - exponential decay

  - as b gets smaller (closer to 0), the graph decreases more rapidly (steeper)

* b cannot be negative, 0, or 1 or it will not be an exponential function.

13) a) $\boxed{5^{1/2} = \sqrt{50}}$

$\log_5 \sqrt{50} = 1/2$

b) $\log_{11} 1 = 0$

$11^0 = 1$

14) $\log_9 27 = \dfrac{\log 27}{\log 9} = 1.5$

15) $\boxed{5^{x-1} = 100} \Rightarrow \log_5 100 = x-1 \Rightarrow \dfrac{\log(100)}{\log(5)} = x-1$

$2.86135 = x-1$
$+1 \qquad +1$

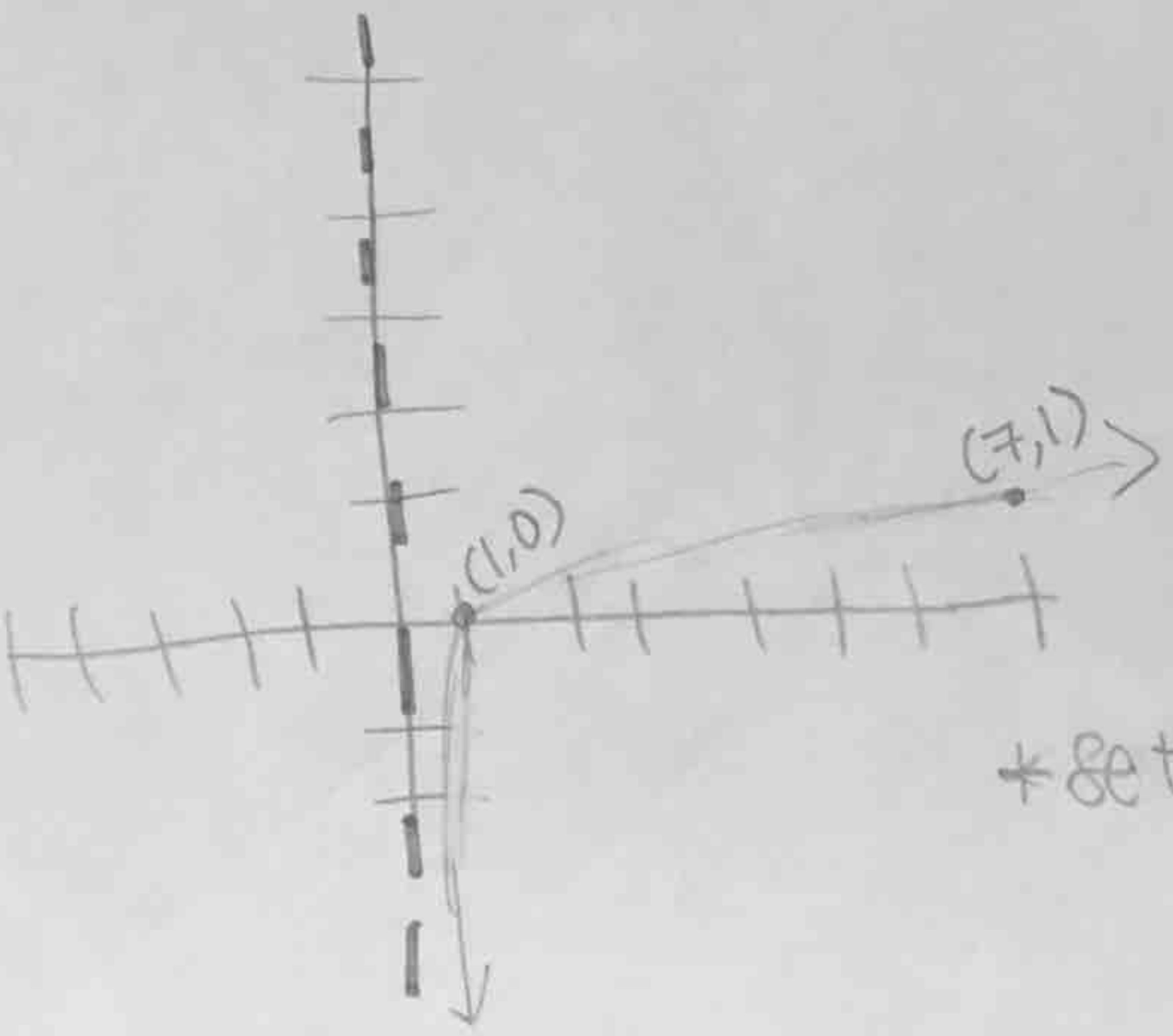
$3.86135 = x$

16) $\log_x 20 = 3 \Rightarrow \sqrt[3]{x^3} = \sqrt[3]{20} \Rightarrow x = \pm 2.7144$

$\boxed{x = 2.7144}$

impossible to have a negative log!

17) $f(x) = \log_7 x$



domain - $(0, \infty)$
range - $(-\infty, \infty)$
y-intercept - None
asymptote - $x = 0$

* set $x = 1 \rightarrow f(1) = \log_7 1 = \dfrac{\log(1)}{\log(7)} = 0$

$(1,0)$

* set $x = 7 \rightarrow f(7) = \log_7 7 = \dfrac{\log(7)}{\log(7)} = 1$

$(7,1)$

18) $A = P\left(1 + \frac{r}{n}\right)^{nt}$

A = ending amount
r = interest rate
p = beginning amount
n = # of compoundings a year
t = total number of years

P = beginning amount = 7500
r = interest rate = 5% = 0.05
n = # of compoundings a year = monthly = 12

$$A = 7500\left(1 + \frac{.05}{12}\right)^{12t}$$
$$= 7500\,(1 + .004167)^{12t}$$
$$= 7500\,(1.004167)^{12t}$$

double money = multiplys by 2

$$\frac{15000}{7500} = \frac{7500(1.004167)^{12t}}{7500}$$
$$2 = (1.004167)^{12t}$$
$$\boxed{(1.004167)^{12t} = 2}$$

rewrote to rewrite as log

$$\log_{1.004167} 2 = 12t$$

$$\frac{\log(2)}{\log(1.004167)} = 12t$$
$$\frac{166.688}{12} = \frac{12t}{12}$$
$$\boxed{13.89 = t}$$

After about 13.89 years, you would double your money.

19) 400mg, decreasing by 29%, compounded continuously

(ibuprofen)

must use

$$f(t) = 400 \cdot e^{-.29t}$$

$$f(6) = 400 \cdot e^{-.29(6)}$$

$$= \boxed{70.21 \text{ mg}}$$

about 70.21mg of Ibuprofen is left after 6 hours

$$f(t) = y_0 \cdot e^{kt}$$

t ← time (variable)

k: interest/decline rate

y_0: starting value

e: continuously