May 30, 2018

Exponential Growth and Interest

Objective: Use the interest formula to model situations involving interest

*Recall the exponential model...

$$f(x) = a(b)^x$$

start value

* If $b>1$, it's exponential growth

$$f(x) = a(1+c)^x$$

growth rate

* If $0<b<1$, it's exponential decay

$$f(x) = a(1-c)^x$$

decay rate

WHAT IS INTEREST?!? you earn interest (more $) when you lend money or deposit funds into an interest bearing bank account such as a savings account or a certificate of deposit (CD)

example) invested $100 at 8% per year.

growth rate

$8\% \xrightarrow{\div 100} 0.08$

Percent → decimal

+ Always exponential growth!

$$f(x) = a(1+c)^x$$
$$= 100(1+0.08)^x$$
$$= 100(1.08)^x$$

- How much money will I have in 20 years if no $ is added or taken out?

$X=20 \rightarrow f(20) = 100(1.08)^{20}$
$= \$466.09$ <- after 20 years

- How many years will it take to double the money in the account?

$f(x) = 200 \longrightarrow \dfrac{100(1.08)^x}{100} = \dfrac{200}{100}$ <- get exponent alone

$1.08^x = 2$ <— change into log to solve for x!

$\log_{1.08} 2 = x$

$\dfrac{\log(2)}{\log(1.08)} = x$

$9 = x$

It will take 9 years to double the money!

Interest formula -

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

amount $ in account

start value/amount deposited

interest rate (decimal)

# of times interest is compounded per year

time (years)

* Interest isn't usually taken out only once per year...
It can be compounded...

- biannually - twice a year ($n=2$)
- quarterly - 4x a year ($n=4$)
- monthly - 12x a year ($n=12$)
- daily - 365x a year ($n=365$)
- hourly - 8760x a year ($n=8760$)
- instantaneously - very very small piece of time
(we will learn tomorrow)

example) investing \$1000 at 12% interest

$$\underset{P}{\$1000} \qquad r = 12\% = 0.12$$

- compounded once per year? ($n=1$)

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$
$$= 1000\left(1+\frac{0.12}{1}\right)^{1t}$$
$$= 1000(1.12)^{t}$$

after 1 year?
$t=1$
$1000(1.12)^{1}$
$= \$1120$

- compounded biannually? ($n=2$)

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$
$$= 1000\left(1+\frac{0.12}{2}\right)^{2t}$$
$$= 1000(1+0.06)^{2t}$$
$$= 1000(1.06)^{2t}$$

after 1 year?
$t=1$
$1000(1.06)^{2(1)}$
$= \$1123.60$

- compounded quarterly? $(n = 4)$

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$
$$= 1000\left(1 + \frac{0.12}{4}\right)^{4t}$$
$$= 1000\left(1 + 0.03\right)^{4t}$$
$$= 1000\left(1.03\right)^{4t}$$

after 1 year?
$t = 1$

$$1000(1.03)^{4(1)}$$
$$= \$1125.50$$

*** The more often its compounded each year the more you make!!!

More examples (If needed) -

ex 1) Investing (\$1000 at 12%) interest. How much will I have in 1 year if its compounded daily?

$P = 1000$, $r = 12\% = 0.12$, $t = 1$, $n = 365$

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$
$$= 1000\left(1 + \frac{0.12}{365}\right)^{365(1)}$$
$$= \$1127.47$$

ex 2) How much will a \$4000 investment be worth if its invested at 8% compounded quarterly for 5 years?

$P = 4000$, $r = 8\% = 0.08$, $t = 5$, $n = 4$

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$
$$= 4000\left(1 + \frac{0.08}{4}\right)^{4(5)}$$
$$= \$5943.79$$

ex 3) How long will it take a \$7000 investment, compounded monthly at 9% interest to double?

(P=7000)

$n=12$ $r=9\%=0.09$ $A=14000$

$$A=P\left(1+\frac{r}{n}\right)^{nt}$$

$$14000=7000\left(1+\frac{0.09}{12}\right)^{12t}$$

$$\frac{14000}{7000}=\frac{7000(1.0075)^{12t}}{7000}$$

$$2=(1.0075)^{12t}$$

7000 must be multiplied by 2 to double!

$$\boxed{(1.0075)^{12t}=12}$$

rewrite to make
↳ to change into a logarithm

$$\log_{1.0075}12=12t$$

$$\frac{\log(12)}{\log(1.0075)}=12t$$

$$\frac{332.56}{12}=\frac{12t}{12}$$

$$27.71=t$$

It will take 27.7 years to double the money!