Polynomial function Unit test Study Guide / Review

Long division -

Polynomial Long Division:

Step 1: Make sure the polynomial is written in descending order. If any terms are missing, use a zero to fill in the missing term (this will help with the spacing).

Step 2: Divide the term with the highest power inside the division symbol by the term with the highest power outside the division symbol.

Step 3: Multiply (or distribute) the answer obtained in the previous step by the polynomial in front of the division symbol.

Step 4: Subtract and bring down the next term.

Step 5: Repeat Steps 2, 3, and 4 until there are no more terms to bring down.

Step 6: Write the final answer. The term remaining after the last subtract step is the remainder and must be written as a fraction in the final answer.

ex) $\dfrac{9u^4+6u^3+0u^2+4u+4}{3u^2+2u+2} \Rightarrow$

$$\begin{array}{r l}
 & 3u^2-2 \quad R\ 8u+8 \\
3u^2+2u+2 \,\big) & 9u^4+6u^3+0u^2+4u+4 \\
3u^2(3u^2+2u+2) \rightarrow & -(9u^4+6u^3+6u^2) \\
 & -6u^2+4u+4 \\
-2(3u^2+2u+2) \longrightarrow & -(-6u^2-4u-4) \\
 & 8u+8
\end{array}$$

Want to eliminate largest degree term!

$$= \boxed{3u^2-2+\frac{8u+8}{3u^2+2u+2}}$$

Synthetic division -

Polynomial Synthetic Division:

Step 1 : To set up the problem, first, set the denominator equal to zero to find the number to put in the division box. Next, make sure the numerator is written in descending order and if any terms are missing you must use a zero to fill in the missing term, finally list only the coefficient in the division problem.

Step 2 : Once the problem is set up correctly, bring the leading coefficient (first number) straight down.

Step 3 : Multiply the number in the division box with the number you brought down and put the result in the next column.

Step 4 : Add the two numbers together and write the result in the bottom of the row.

Step 5 : Repeat steps 3 and 4 until you reach the end of the problem.

Step 6 : Write the final answer. The final answer is made up of the numbers in the bottom row with the last number being the remainder and the remainder must be written as a fraction. The variables or x's start off one power less than the original denominator and go down one with each term.

ex) $\dfrac{x^3-3x^2+5x-4}{x+2}$

$x+2=0$
$-2\quad -2$
$x=-2$

x^3-3x^2+5x-4

$$\begin{array}{r|rrrr} \boxed{-2} & 1 & -3 & 5 & -4 \\ & \downarrow & -2 & 10 & -30 \\ \hline & 1 & -5 & 15 & -34 \end{array}$$

remainder

$\boxed{x^2-5x+15-\dfrac{34}{x+2}}$

"add straight down"
"multiply last row by special # and write in next column"

ex) $\dfrac{6x^2+7x+2}{3x+2}$

$3x+2=0$
$-2\quad -2$
$\dfrac{3x}{3}=\dfrac{-2}{3}$
$x=-\frac{2}{3}$

$6x^2+7x+2$

$$\begin{array}{r|rrr} \boxed{-\frac{2}{3}} & 6 & 7 & 2 \\ & \downarrow & -4 & -2 \\ \hline & 6 & 3 & 0 \end{array}$$

$\boxed{\dfrac{6x+3}{3}=2x+1}$

Factor and Remainder Theorem -

Remainder theorem - The remainder of a quotient between a polynomial equation and a number is the solution to the equation

$f(5) = -x^4 + 3x^3 + 6x + 3$
$= -(5)^4 + 3(5)^3 + 6(5) + 3$
$= -217$

$-x^4 + 3x^3 + 0x^2 + 6x + 3$

$\boxed{5}$	-1	3	0	6	3
	↓	-5	-10	-50	-220
	-1	-2	-10	-44	-217

factor theorem - A polynomial has r as a root (or x-r as a factor) if and only if $P(r) = 0$ or if when you synthetically divide, the remainder is 0.

ex) is $x-5$ a factor of $f(x) = x^3 - 5x^2 + 2x - 10$.

Basically... $\frac{x^3 - 5x^2 + 2x - 10}{x-5}$, does it come out even?

$x - 5 = 0$
$+5 \quad +5$
$x = 5$

$\boxed{5}$	1	-5	2	-10
	↓	5	0	10
	1	0	2	0

← remainder is 0! it is a factor!

$= x^2 + 2$

equation - $(x-5)(x^2+2) = x^3 - 5x^2 + 2x - 10$

ex) is $x+1$ a factor of $f(x) = x^3 - 3x^2 - 2x - 2$?

$x + 1 = 0$
$-1 \quad -1$
$x = -1$

$\boxed{-1}$	1	-3	-2	-2
	↓	-1	4	-2
	1	-4	2	-4

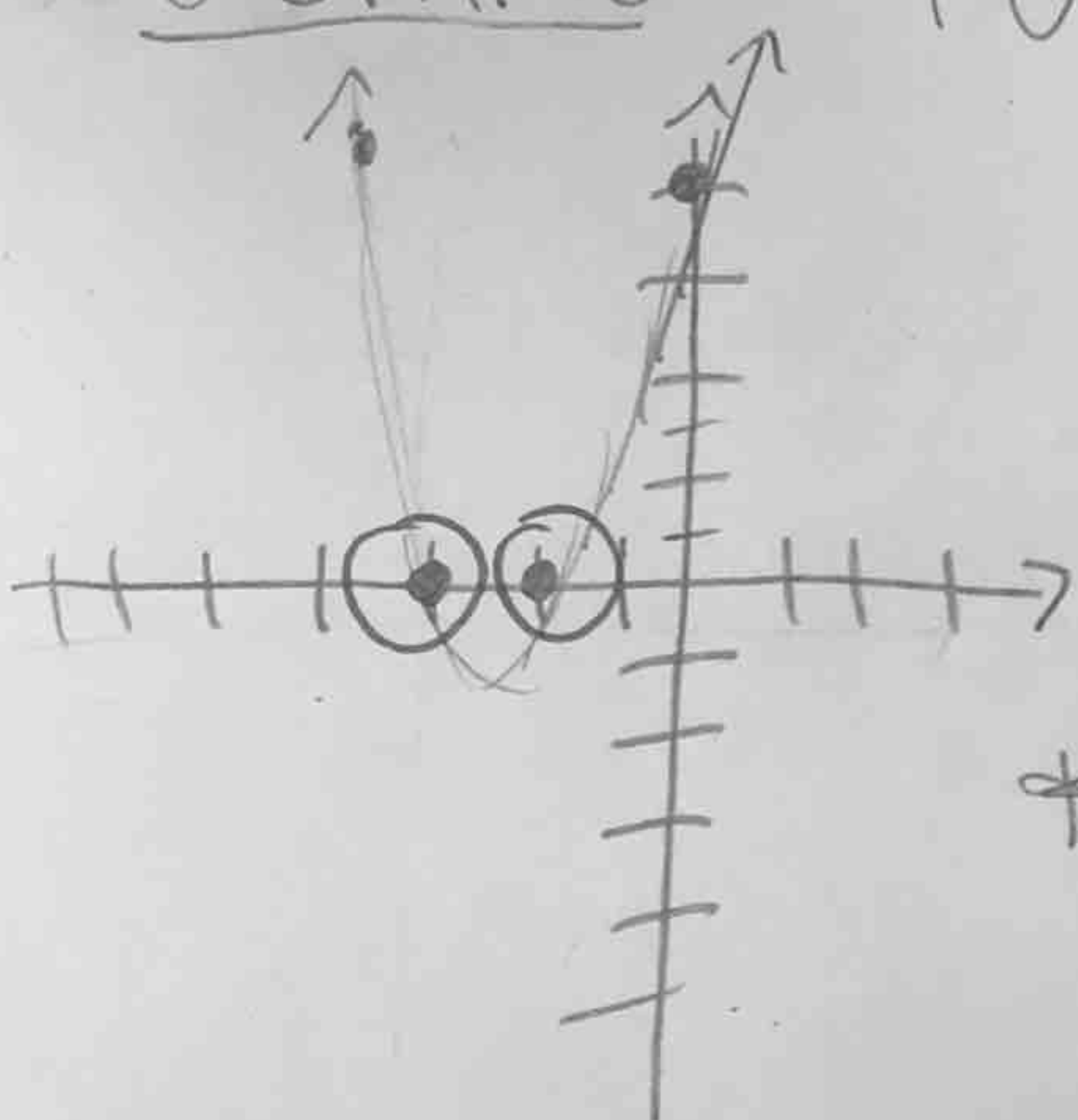
← remainder is NOT 0, it is not a factor!!

Polynomial equations-

<u>factor</u>- Numbers or expressions that we can multiply together to get another number or expression.

<u>root</u>- Where a function equals zero (also called zeros, solutions, x-intercept)

<u>example</u>- $f(x) = x^2 + \underset{2+3}{5}x + \underset{2\cdot 3}{6}$ <— expanded polynomial



$= \underbrace{(x+2)(x+3)}_{\text{factors}}$ <— factored polynomial

*set each factor pair equal to 0 to get roots!

$$\begin{array}{r} x+2=0 \\ -2\ \ -2 \\ \hline \underline{x=-2} \end{array} \qquad \begin{array}{r} x+3=0 \\ -3\ \ -3 \\ \hline \underline{x=-3} \end{array}$$

<u>example</u>- roots are 2, -4, and 3. write a polynomial equation. <u>Work Backwards!</u>

$$\begin{array}{r} x=2 \\ -2\ \ -2 \\ \hline x-2=0 \end{array} \qquad \begin{array}{r} x=-4 \\ +4\ \ +4 \\ \hline x+4=0 \end{array} \qquad \begin{array}{r} x=3 \\ -3\ \ -3 \\ \hline x-3=0 \end{array}$$

*multiply factors together to get polynomial

$\underbrace{(x-2)(x+4)}(x-3)$ <— box method to expand!!

$=(x^2+2x-8)(x-3)$

$= \boxed{x^3 - x^2 - 14x + 24}$

	x	-2
x	x^2	$-2x$
4	$4x$	-8

	x^2	$2x$	-8
x	x^3	$2x^2$	$-8x$
-3	$-3x^2$	$-6x$	24

* If you are given an imaginary or $\sqrt{\ }$ root, you **must** make sure you have the + and − root!!!

ex) roots are 2, $3i$, write a polynomial equation

must also include $-3i$, even if not listed!!!

$$x = 2 \qquad x = 3i \qquad x = -3i$$
$$-2 \ \ -2 \qquad -3i \ \ -3i \qquad +3i \ \ +3i$$
$$x - 2 = 0 \qquad x - 3i = 0 \qquad x + 3i = 0$$

$$(x-2)\underbrace{(x-3i)(x+3i)}$$
$$= (x-2)(x^2+3^2)$$
$$= (x-2)(x^2+9)$$
$$= \boxed{x^3 - 2x^2 + 9x - 18}$$

	x	-2
x^2	x^3	$-2x^2$
9	$9x$	-18

RULE -
$(a+bi)(a-bi) = a^2 + b^2$

ex) roots are 4 and $\sqrt{6}$

must also include $-\sqrt{6}$, even if not listed!!!

$$x = 4 \qquad x = \sqrt{6} \qquad x = -\sqrt{6}$$
$$-4 \ \ -4 \qquad -\sqrt{6} \ \ -\sqrt{6} \qquad +\sqrt{6} \ \ +\sqrt{6}$$
$$x - 4 = 0 \qquad x - \sqrt{6} = 0 \qquad x + \sqrt{6} = 0$$

$$(x-4)(x-\sqrt{6})(x+\sqrt{6})$$
$$= (x-4)(x^2-6)$$
$$= \boxed{x^3 - 4x^2 - 6x + 24}$$

	x^2	-6
x	x^3	$-6x$
-4	$-4x^2$	24

RULE -
$(a+\sqrt{b})(a-\sqrt{b}) = a^2 - b$

Solving Polynomial equations -

*remember, the degree of the polynomial tells you how many roots!

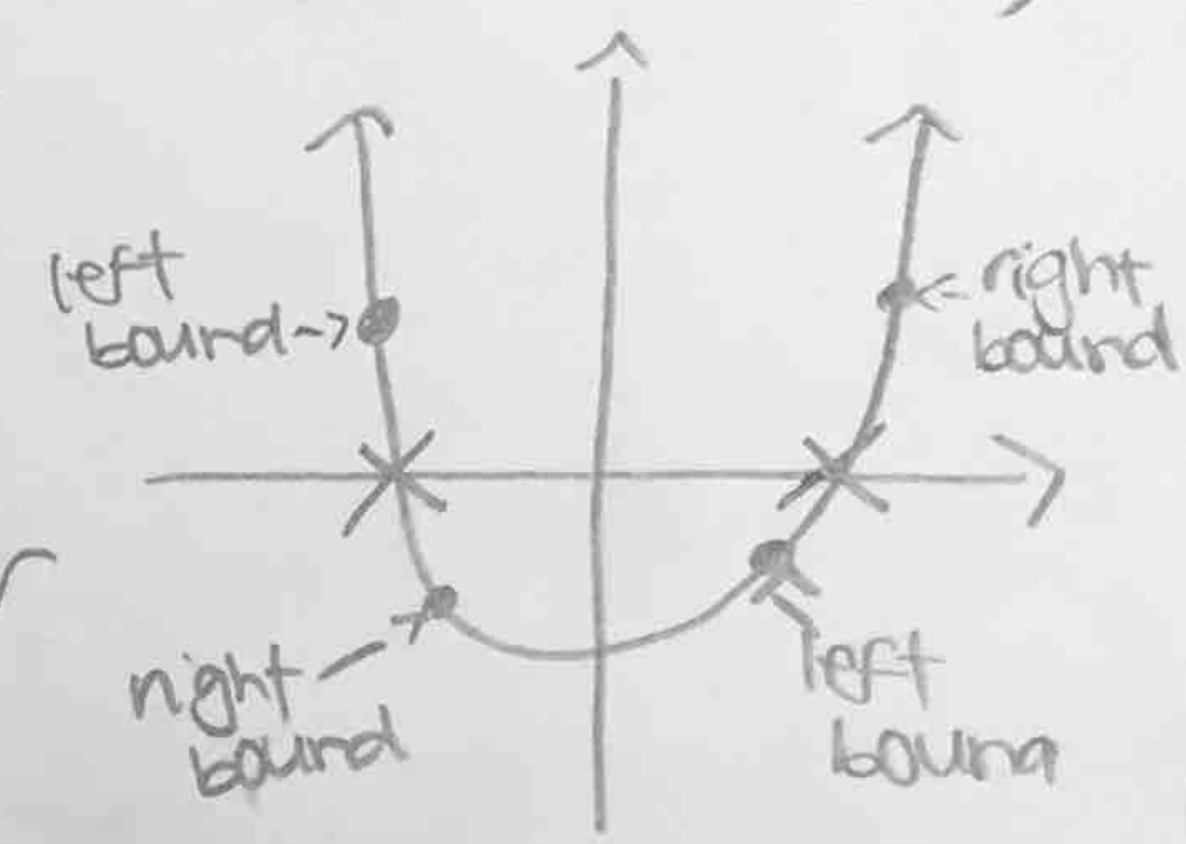
* If asked to find zeros/roots (rational and irrational) of a polynomial, steps:

1) Type into calculator to see all possible rational roots. (where it crosses the x-axis and/or where $y=0$)

   - $y=$ (top left)
   - Type in polynomial equation into y_1
   - graph (top right) <— to look at graph

     and/or

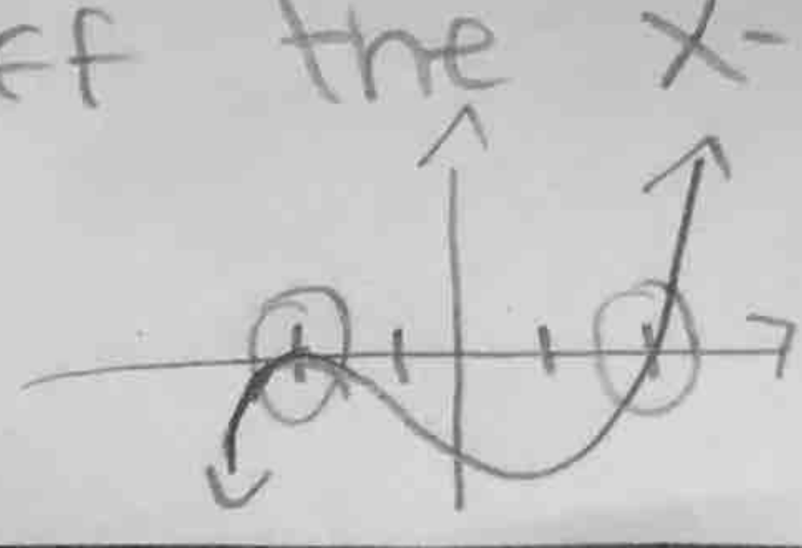
   - 2nd, graph (top right) <— to look at data table (find where $y=0$)
   - If cannot see x-intercepts
     - 2nd, trace
     - 2: zero
     - left bound, enter
     - right bound, enter
     - enter



*If it's fractional and cannot accurately place x-intercept. Must find

- factors of constant ($\pm p$)
- factors of leading coefficient ($\pm q$)
  (# in front of highest degree variable)
- Possible roots are $\pm \frac{p}{q}$
- estimate which would make the most sense.

*If it bounces off the x-axis, it's a root twice!



roots $= -2, 2$
twice

2) If only find a few roots from graph, must synthetically divide down to a quadratic ($ax^2+bx+c=0$) or a linear ($mx+b=0$) to solve!

- Can solve linears through solving for x (multistep equations)
- Can solve quadratics by factoring, quadratic formula, or completing the square.

examples - find all zeros/rational and irrational roots.

$f(x)=x^{4}-1$ ← 4 roots ← from calculator (but need 4 roots!!!)

$x=1,-1$

* Must synthetically divide using roots we know to end up with a quadratic.

$x^4+0x^3+0x^2+0x-1$

$$\begin{array}{c|ccccc} \boxed{1} & 1 & 0 & 0 & 0 & -1 \\ & \downarrow & 1 & 1 & 1 & 1 \\ \hline & 1 & 1 & 1 & 1 & 0 \end{array}$$

$x^3+x^2+x+1=0$

$$\begin{array}{c|cccc} \boxed{-1} & 1 & 1 & 1 & 1 \\ & \downarrow & -1 & 0 & -1 \\ \hline & 1 & 0 & 1 & 0 \end{array}$$

Solve to find other 2 roots! → $x^2+1=0$

$$\begin{aligned} x^2+1&=0 \\ -1\quad&\;\;-1 \\ \hline x^2&=-1 \\ \sqrt{x^2}&=\pm\sqrt{-1} \\ x&=\pm i \end{aligned}$$

$\boxed{\text{roots}=1,-1,\pm i}$

* If asked to put in factored form:

$(x-1)(x+1)(x+i)(x-i)$

4 roots

ex) $f(x) = x^{4} - 4x^3 - 5x^2 + 12x - 4 = 0$ ← from calculator $x = -2, 1$

(but need 4 roots!!)

* Must synthetically divide using roots we know to end up with a quadratic.

$x^4 - 4x^3 - 5x^2 + 12x - 4$

$\boxed{-2}$	1	−4	−5	12	−4
	↓	−2	12	−14	4
	1	−6	7	−2	0
	↓	↓	↓	↓	
$\boxed{1}$	1	−6	7	−2	
	↓	1	−5	2	
	1	−5	2	0	

$x^2 - 5x + 2 = 0$ ← $a = 1$, $b = -5$, $c = 2$

Quadratic formula -

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{5 \pm \sqrt{(-5)^2 - 4(1)(2)}}{2(1)} = \frac{5 \pm \sqrt{25 - 8}}{2} = \frac{5 \pm \sqrt{17}}{2}$$

$$\boxed{\text{roots} = -2, 1, \frac{5 \pm \sqrt{17}}{2}}$$

* If asked to put in factored form:

$$(x+2)(x-1)\left(x - \frac{5+\sqrt{17}}{2}\right)\left(x - \frac{5-\sqrt{17}}{2}\right)$$

Graphs of Polynomial functions -

- Positive, even - ← end behavior, both up

ex) x^2, x^4, x^6, ...

ex) $5x^2 + x - 3$

ex) $4x^{12} - x^5 + x + 3$

- negative, even - ← end behavior, both down

ex) $-x^2$, $-x^4$, $-x^6$, ...

ex) $-12x^4 + x^3 - 5$

ex) $-5x^{24} + x^{13} - x$

* Only care about highest degree term and the sign in front!

* may have multiple bumps in between, but the end behavior stays the same!

- Positive, odd - ← end behavior, right up, left down

ex) x^3, x^5, x^7, ...

ex) $x^7 + x^4 + x^2 + 3$

ex) $x^{23} - x + 2$

- negative, odd - ← end behavior, left up, right down

ex) $-x^3$, $-x^5$, $-x^7$

ex) $-x^3 + x^2 - 5x + 2$

ex) $-x^{43} - x^{20} + x^{14} - x + 10$

*When sketching graphs of polynomials...

<u>must</u> include:

* The zeros of the polynomial (x-intercepts)
* The y-intercept
* Appropriate end behavior (↺, ↻, ∿, ∿)

- Can get a good idea from calculator!

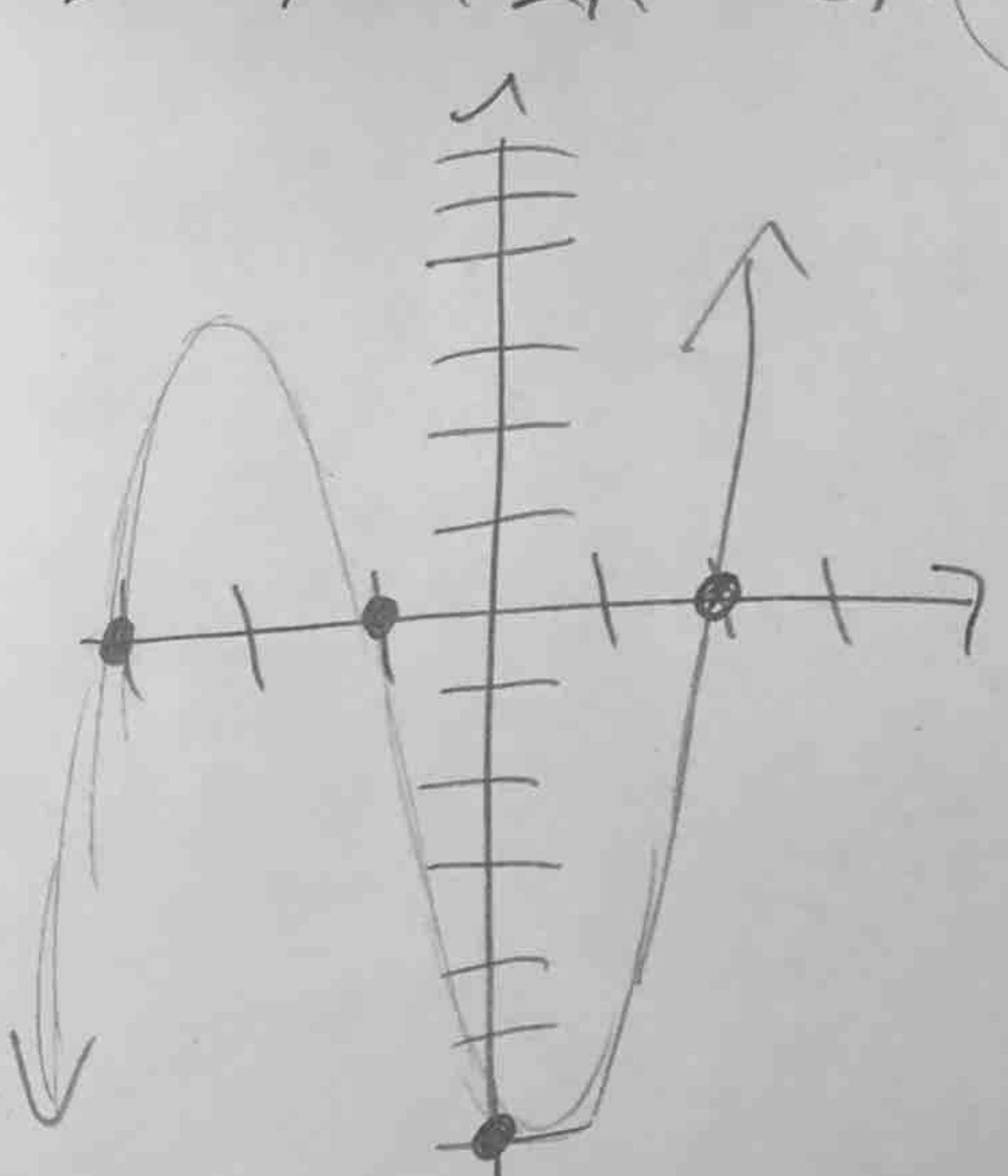
ex) $f(x) = x^3 + 2x^2 - 5x - 6$

y-intercept - $y = -6$

x-intercepts - (from calculator)

$x = 2, -1, -3$

general shape - positive, odd



ex) $f(x) = (x+4)(x-1)(6x-2)$

y-intercept - $y = (4)(-1)(-2) = 8$

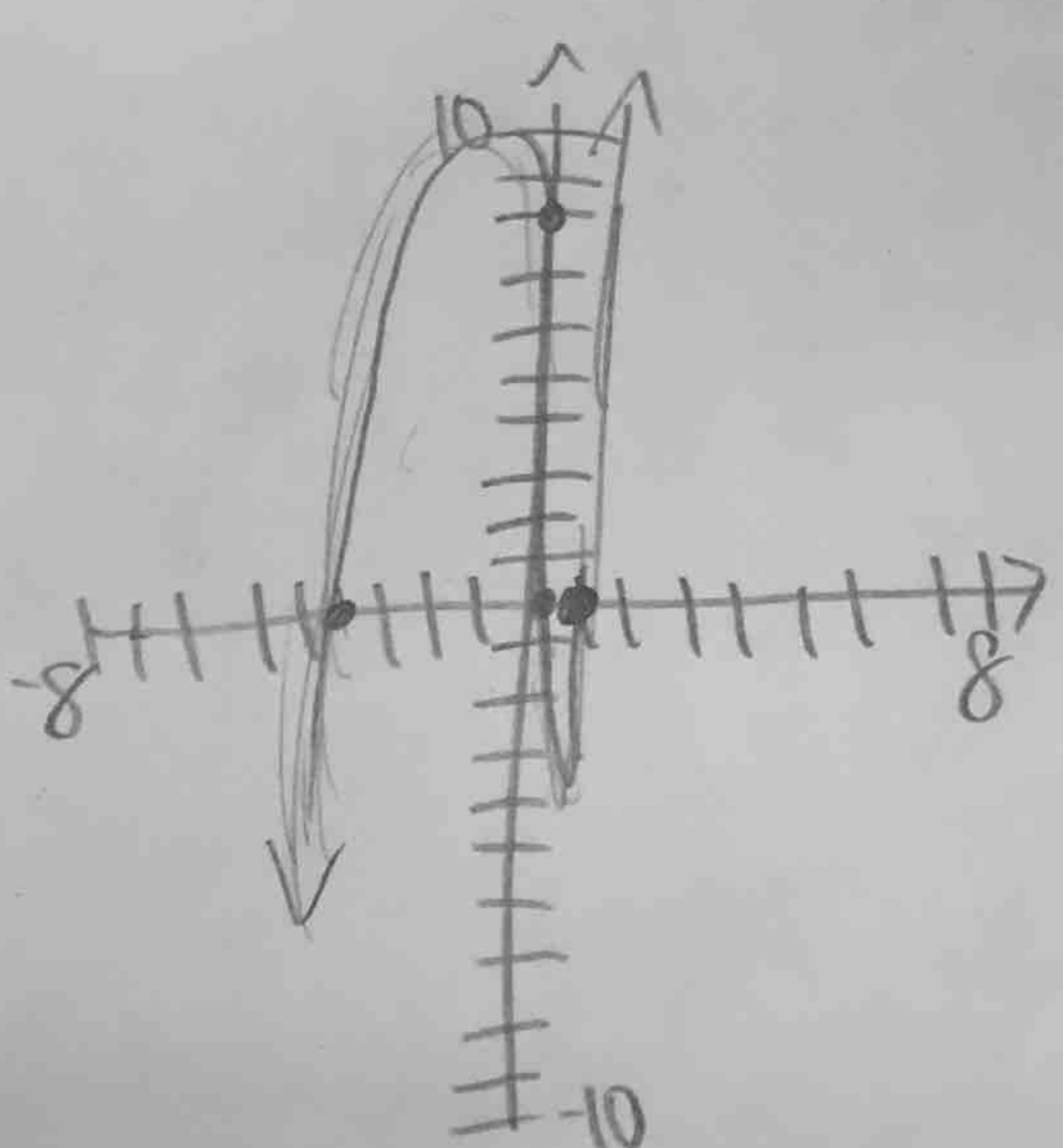
x-intercepts - $x = -4, 1, \frac{1}{3}$

$x+4=0$
$-4 \quad -4$
$x=-4$

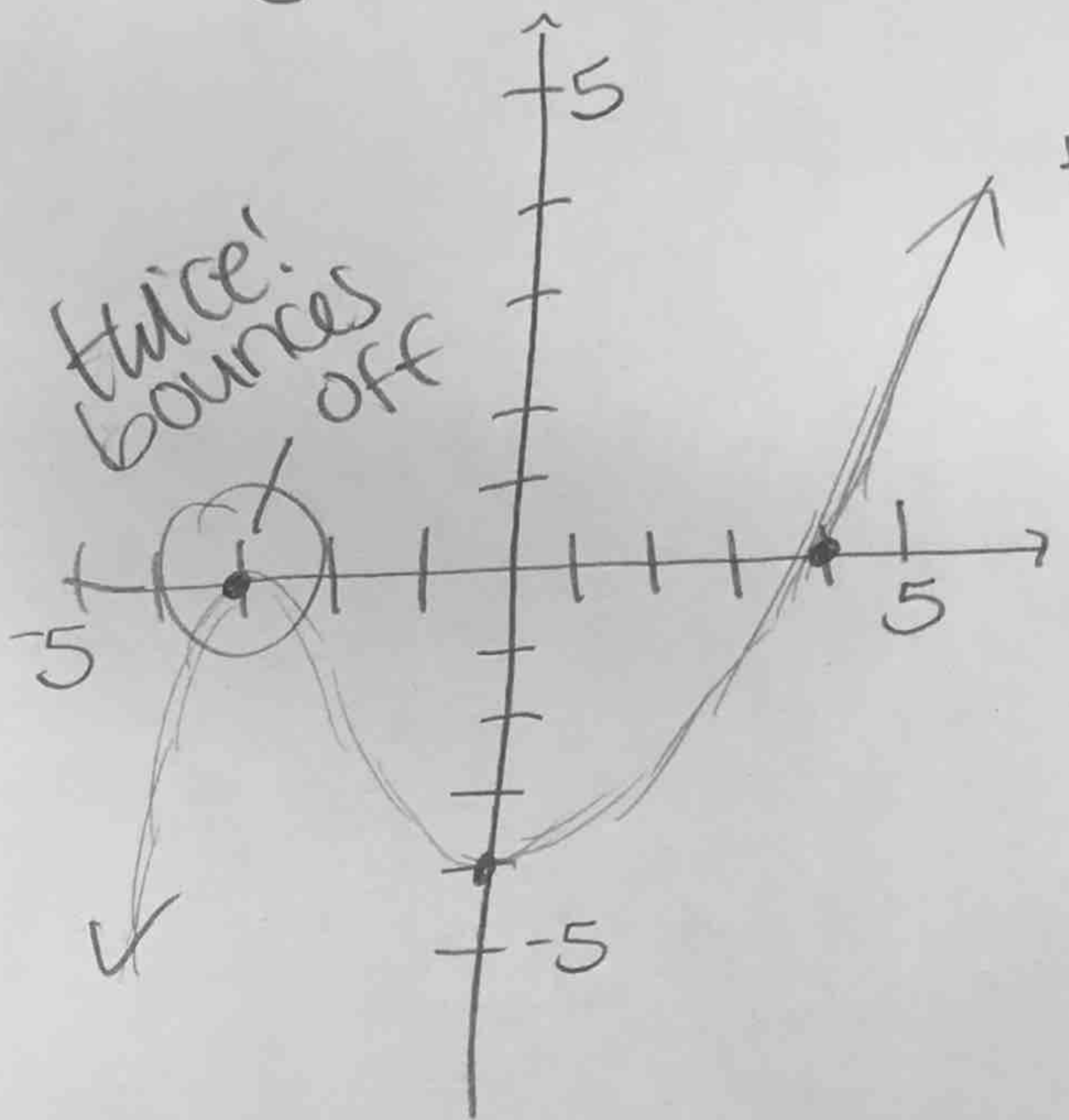
$x-1=0$
$+1 \quad +1$
$x=1$

$6x-2=0$
$+2 \quad +2$
$\frac{6x}{6}=\frac{2}{6}$
$x=\frac{1}{3}$

general shape - positive, odd



ex) Based on the graph, write a potential Polynomial equation:



1) find roots/x-intercepts -

twice! $\rightarrow$ $x = -3$

$\quad +3 \quad +3$

$x+3=0$

$x = 4$

$-4 \quad -4$

$x-4=0$

2) write possible equation

$f(x) = (x+3)^2(x-4)$

3) equation y-int $= (3)^2(-4)$
$= -36$

graph y-int $= -4$

4) If equation and graph y-intercepts don't match up must divide

$\dfrac{\text{graph y-int}}{\text{equation y-int}} = \dfrac{-4}{-36} = \dfrac{1}{9}$

to find what to multiply equation by

5) equation -

$\boxed{f(x) = \frac{1}{9}(x+3)^2(x-4)}$

Transformations of Polynomial functions-

Rules:

* $f(x) = (x-h)^3$ ← left or right 'h' units
* $f(x) = x^3 + k$ ← up or down 'k' units
* $f(x) = -x^3$ ← flipped across x-axis

even degree -

* $f(x) = ax^2, ax^4, ax^6, \dots$

right 1, up 'a'
left 1, up 'a'

odd degree -

* $f(x) = ax^3, ax^5, ax^7$

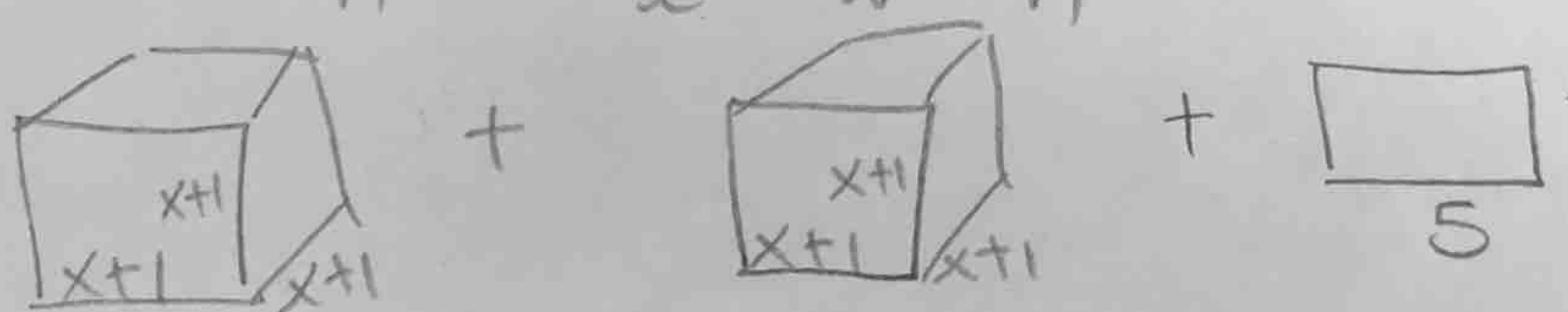
right 1, up 'a'
left 1, down 'a'

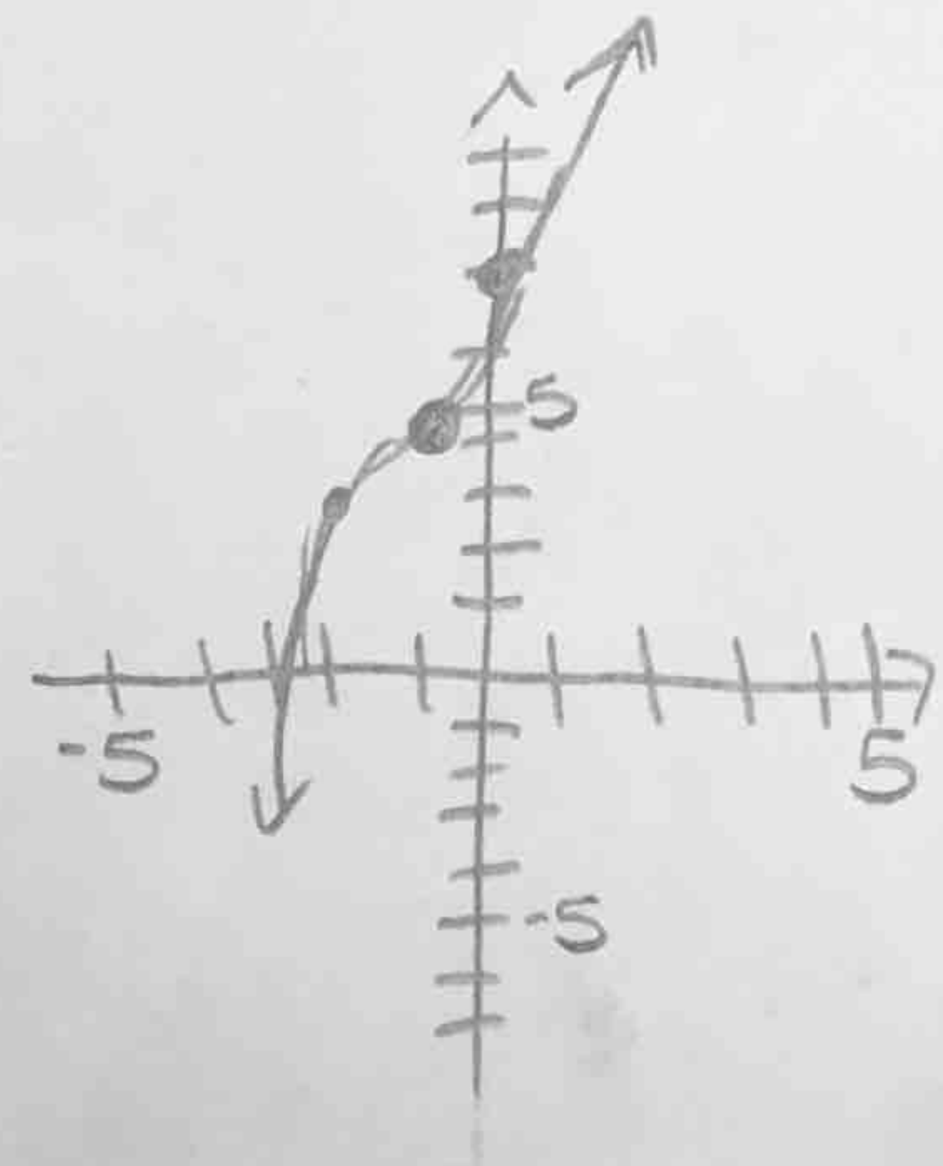
right 1, up 2
left 1, down 2
left 1

ex) $2(x+1)^3 + 5$ ← up 5

$(x+1)(x+1)(x+1) + (x+1)(x+1)(x+1) + 5$

l w h l w h

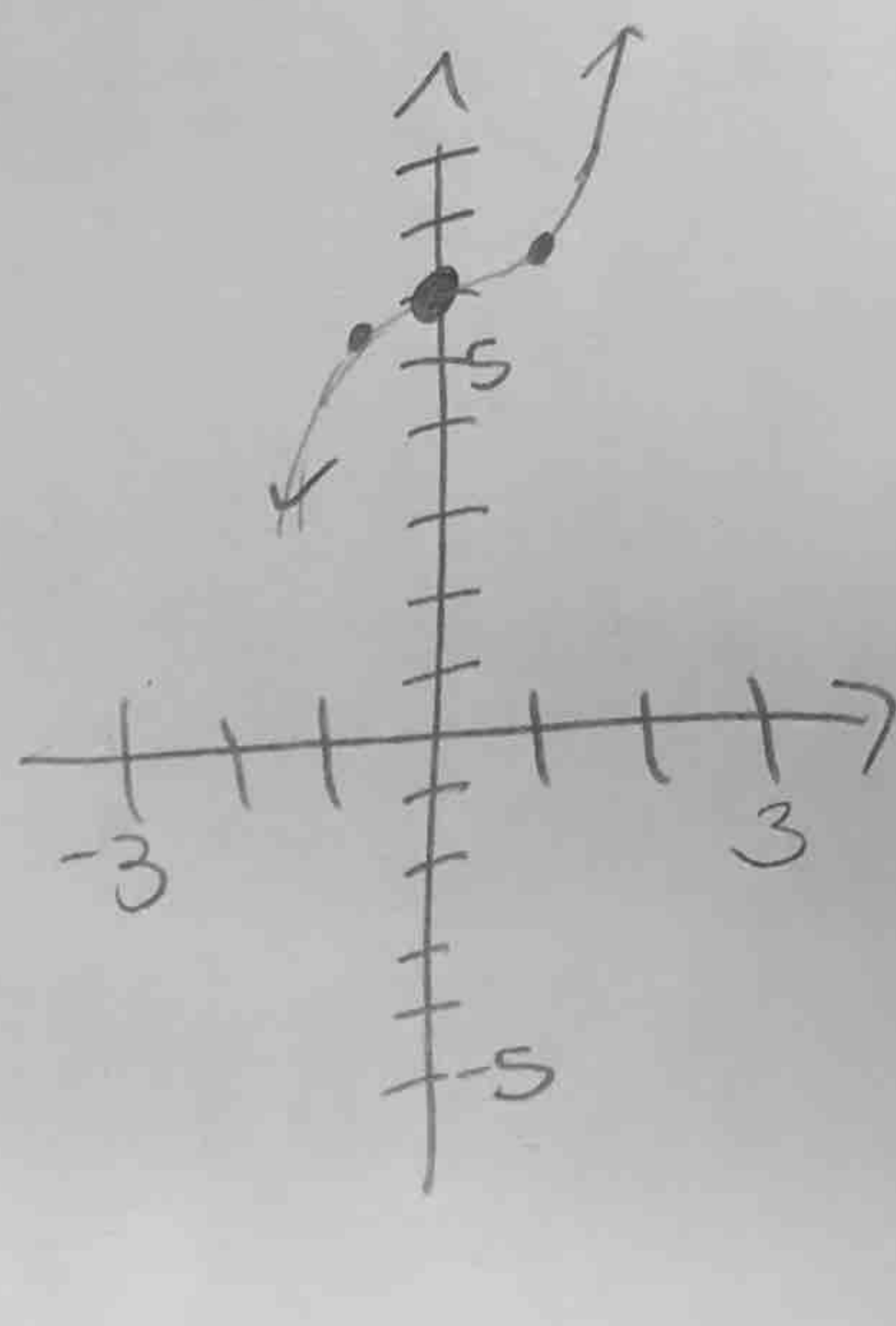


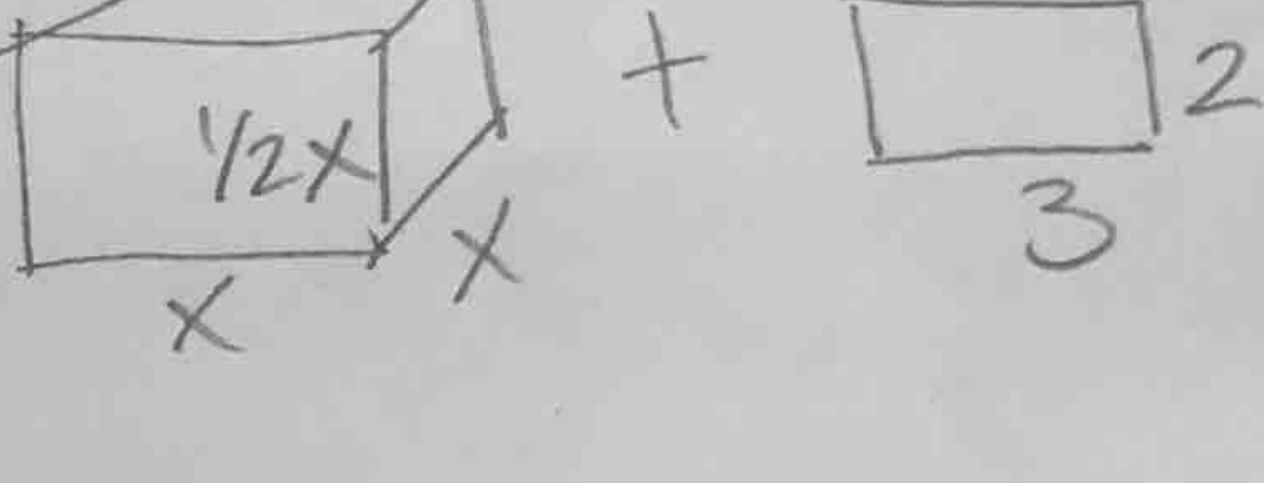


right 1, up 1/2
down 1, down 1/2

ex) $\frac{1}{2}x^3 + 6$ ← up 6

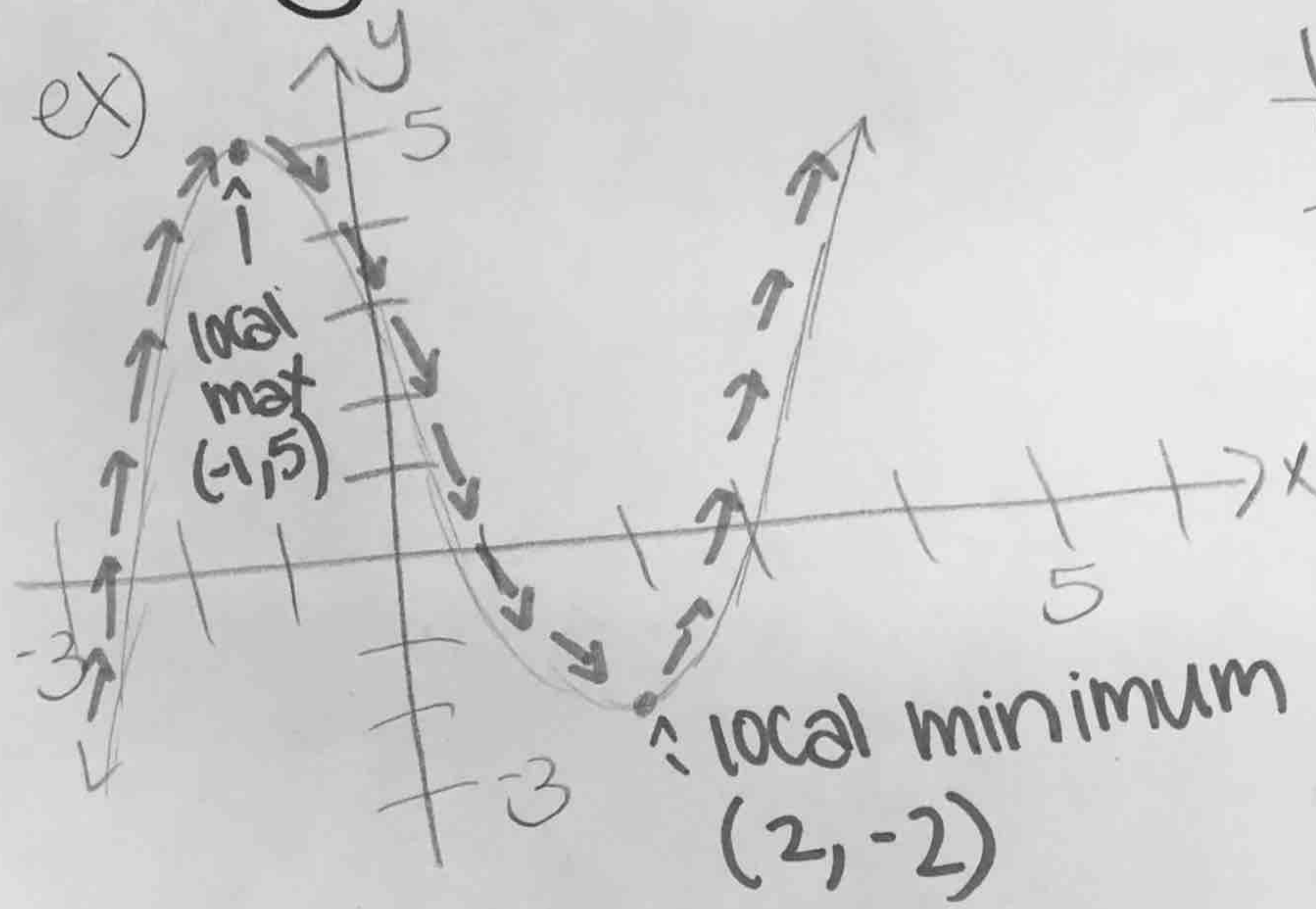
$(\frac{1}{2}x)(x)(x) + 6$





Modeling Volume with Polynomials -

ex)



local max - (-1, 5)

local min - (2, -2)

Increasing (x-values) -

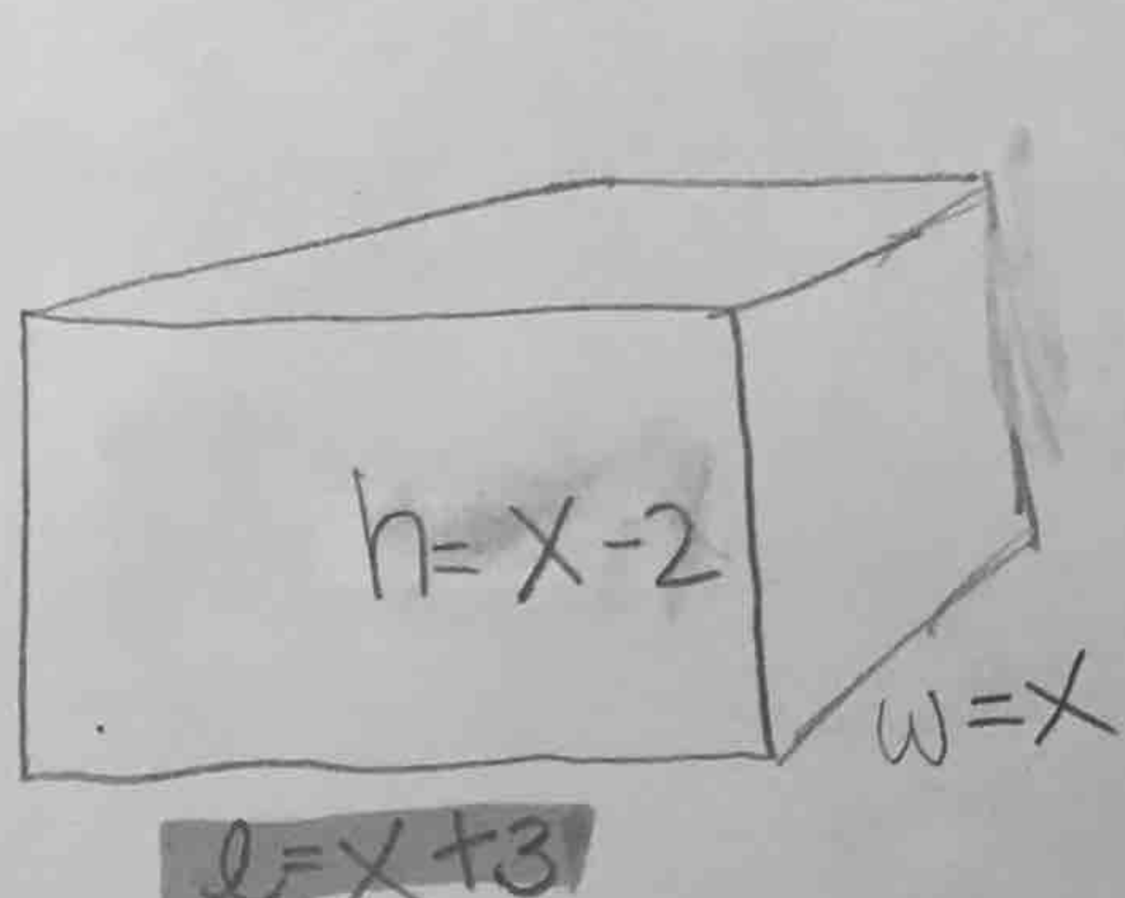
$(-\infty, -1)$, $(2, \infty)$

decreasing (x-values) -

$(-1, 2)$

smallest largest

ex) I have a box whose length is 3in larger than its width, and the height is 2in smaller than the width.



	x^2	$-2x$
x	x^3	$-2x^2$
3	$3x^2$	$-6x$

$$\text{Volume} = l \cdot w \cdot h$$
$$= (x+3)(x-2)(x)$$
$$= (x+3)(x^2-2x)$$
$$= x^3 + x^2 - 6x$$

* If the width is 6in, find the volume -

$$W = x = 6 \Rightarrow V = (6)^3 + (6)^2 - 6(6)$$
$$= 216$$

* If the volume is 56, find the width -

$$V = 56 \Rightarrow \quad 56 = x^3 + x^2 - 6x$$
$$\underline{-56 \qquad\qquad\qquad -56}$$
$$0 = x^3 + x^2 - 6x - 56$$

from calculator

$\boxed{x = 4}$