Polynomial functions Test Review

1) $x^2+3x-1 \overline{)\, x^3+5x^2+4x-1}$

$x+2$

$-(x^3+3x^2-x)$

$2x^2+5x-1$

$-(2x^2+6x-2)$

$-x+1$ remainder

$= \boxed{x+2+\frac{-x+1}{x^2+3x-1}}$

2) $2x-1=0$

$+1 \quad +1$

$\frac{2x}{2}=\frac{1}{2}$

$x = \frac{1}{2} = 0.5$

0.5

x^3	$-3x^2$	$+7x$	-4
1	-3	7	-4
↓	+0.5	+-1.25	+4.6
1	-2.5	5.75	0.6 remainder

$\frac{x^2-2.5x+5.75+\frac{0.6}{2x-1}}{2}$

$\boxed{\frac{x^2}{2}-\frac{2.5x}{2}+\frac{5.75}{2}+\frac{0.6}{2(2x-1)}}$

3) $f(x)=x^4-2x^3+7x-4$

$f(3)=(3)^4-2(3)^3+7(3)-4$

$=81-54+21-4$

$=\boxed{44}$

4) $x+3$ root of $f(x)=2x^3+4x^2-5x+9$?

$x+3=0$

$-3 \quad -3$

$x=-3$

-3

$2x^3$	$+4x^2$	$-5x$	$+9$
2	4	-5	9
↓	+-6	+6	+3
2	-2	-1	12

$2x^2-2x-1+\frac{12}{x+3}$

has a remainder, therefore is NOT a root

OR $X+3=0$, $-3\ -3$, $X=-3$

$f(-3) = 2(-3)^3 + 4(-3)^2 - 5(-3) + 9$
$= -54 + 36 + 15 + 9$
$= \cancel{6}$ ← NOT a root, because it doesn't equal zero.

5) roots: 2, 3, 5i

$X=2$, $-2\ -2$, $X-2=0$

$X=3$, $-3\ -3$, $X-3=0$

$X=5i$, $-5i\ -5i$, $X-5i=0$

$X=-5i$, $+5i\ +5i$, $X+5i=0$ ← must also have neg imaginary root...

$(X-2)(X-3)(X-5i)(X+5i)$

$X-2$ × (X^2-5X+6) × (X^2+25)

$X-2$ × X^2-5X+6:

×	x^2	$-2x$
x	x^3	$-2x^2$
-3	$-3x^2$	$6x$

$= x^4-5x^3+31x^2-125x+150$

$X-5i$ × $X+5i$:

×	x	$-5i$
x	x^2	$-5ix$
$5i$	$5ix$	$-25i^2 = -25(-1) = 25$

OR $(a+bi)(a-bi) = a^2+b^2$
$= a=x,\ b=5 = x^2+5^2$
$= x^2+25$

x^2-5x+6 × x^2+25:

×	x^4	$-5x^3$	$6x^2$
x^2	x^6	$-5x^5$	$6x^4$
$+25$	$25x^4$	$-125x^3$	$150x^2$

3 roots

⑥ $x^3-x^2-14x+24=0$

p: -24: $\pm1, \pm2, \pm3, \pm4, \pm6, \pm8, \pm12, \pm24$

q: -1: ±1

$\frac{p}{q} = \pm1, \pm2, \pm3, \pm4, \pm6, \pm8, \pm12, \pm24$

Zeros / roots / x-intercepts: $x = -4, 2, 3$

$X=-4$, $+4\ +4$, $X+4=0$

$X=2$, $-2\ -2$, $X-2=0$

$X=3$, $-3\ -3$, $X-3=0$

$(X+4)(X-2)(X-3)$ ← switch signs of zeros and add/subtract to X.

7) $2x^4 - 5x^3 + 8x^2 - 15x + 6 = 0$

4 roots.

Zeros / roots / x-intercepts: $x = 2, \frac{1}{2}$

$x = 2$: $-2 \ -2$; $x - 2 = 0$

$x = \frac{1}{2}$: $-\frac{1}{2} \ -\frac{1}{2}$; $x - \frac{1}{2} = 0$

$(x-2)(x-\frac{1}{2})$ ⟵ should have 4 roots....

synthetic division!!

$2x^4 - 5x^3 + 8x^2 - 15x + 6$

[2] | 2 −5 8 −15 6

↓ +4 +−2 +12 +−6

2 −1 6 −3 | 0

$2x^3 - x^2 + 6x - 3$

[½] | 2 −1 6 −3

↓ +1 +0 +3

2 0 6 | 0

what's left over ⟵ $2x^2 + 6$

$x - 2$

x | x^2 | $-2x$

$-\frac{1}{2}$ | $-\frac{1}{2}x$ | 1

$2x^2 + 6 = 0$

$-6 \ -6$

$\frac{2x^2}{2} = \frac{-6}{2}$

$\sqrt{x^2} = \sqrt{3}$

$x = \pm i\sqrt{3}$ ⟵ other 2 roots.

$(x-2)(x-\frac{1}{2})(x+\sqrt{3}i)(x-\sqrt{3}i)$

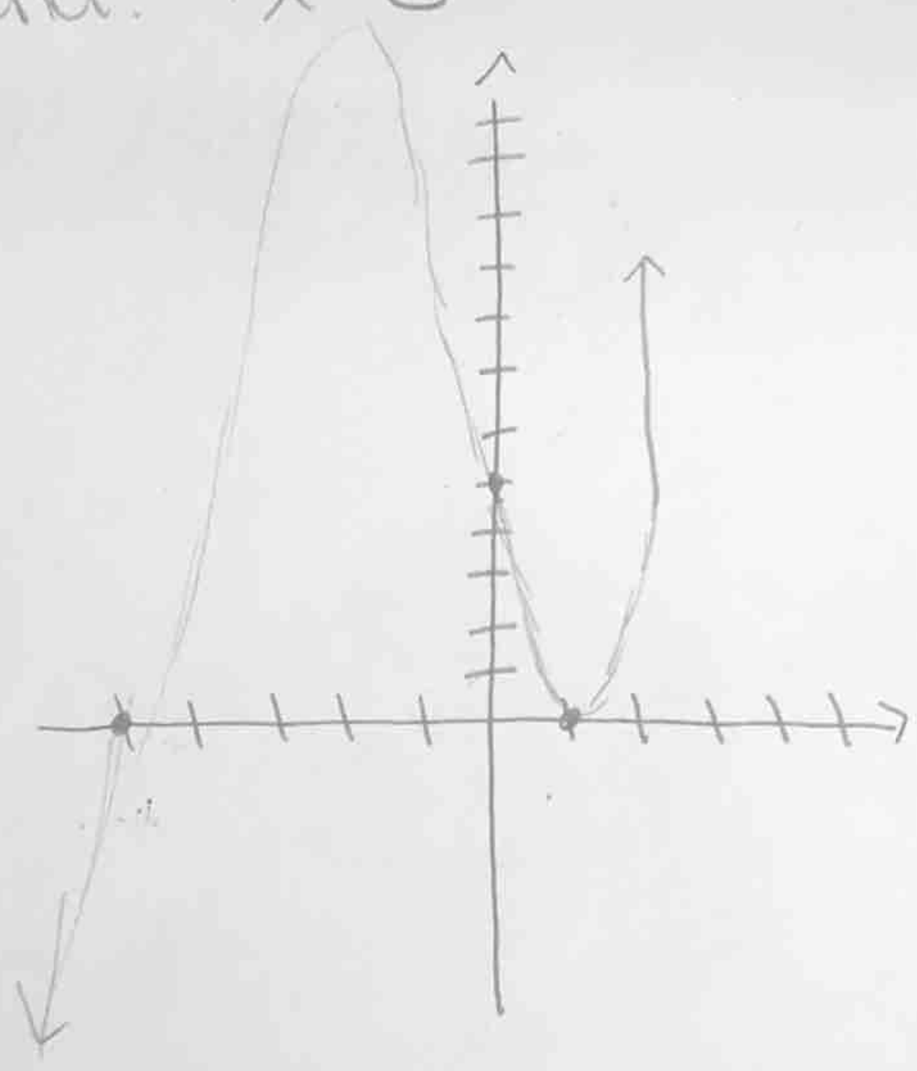
8) $f(x) = x^3 + 3x^2 - 9x + 5$

zeros/x-intercepts: $x = 1$ ← twice! bounces off of x-axis. and -5

$(x-1)^2(x+5)$

y-intercept: constant at the end! $x=5$

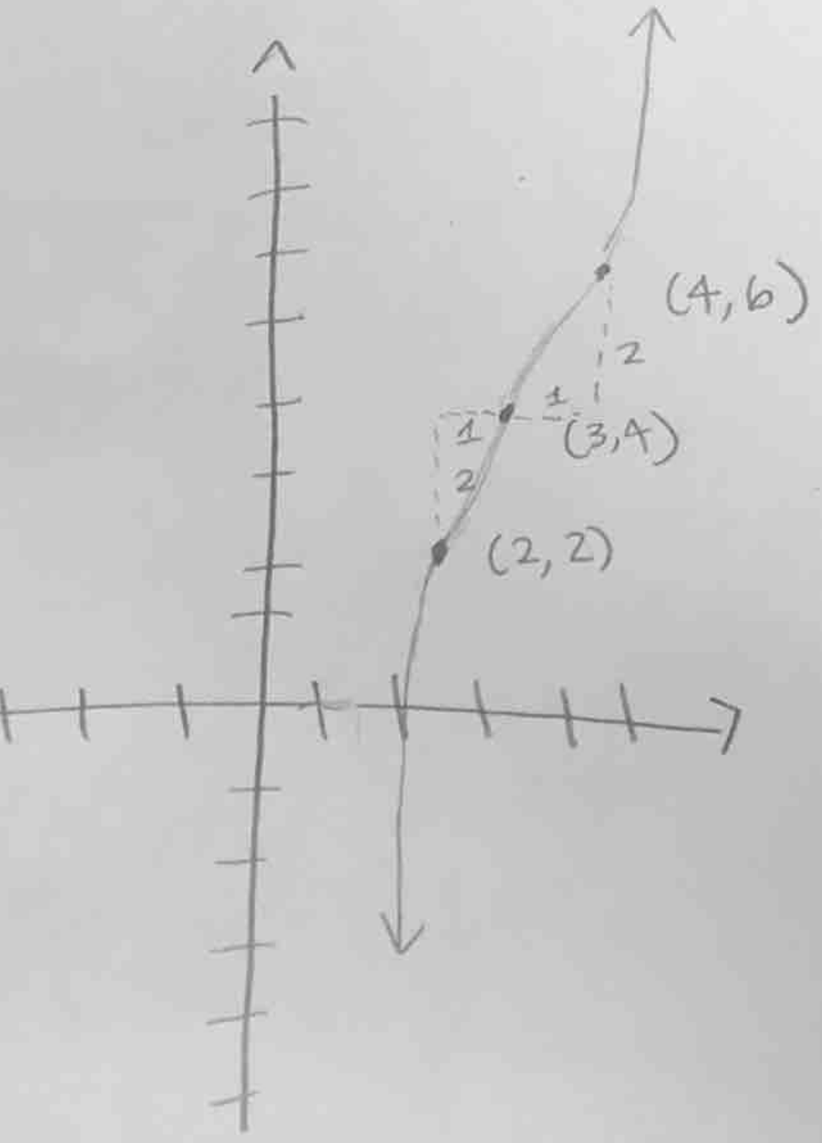
* degree = 3, odd positive ↙↗



9) $-5x^4 - x^2 + 25 = 0$

highest degree = 4

negative even degree ↘↗ both go down.



10) $f(x) = 2(x-3)^3 + 4$

over 1 right, up 2

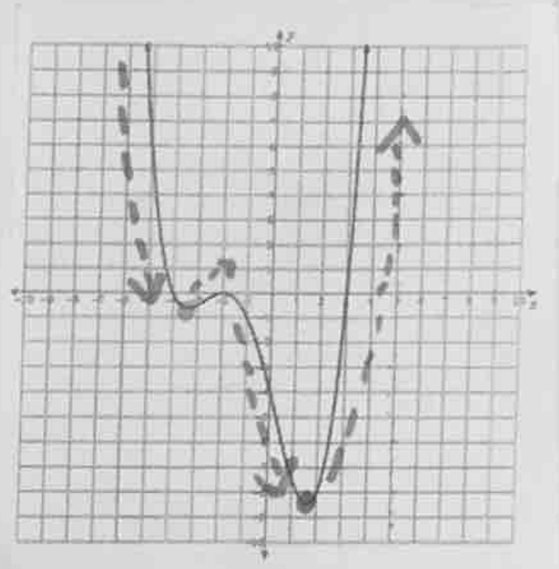
over 1 left, down 2

right 3

up 4

11) • local minimum

• local minimum



∴ interval decreasing
$(-\infty, -3.5), (-2, 1.5)$

∴ interval increasing
$(-3.5, -2), (1.5, \infty)$

12) x-intercepts = $-4, -2, 3$ (−2 twice)

Polynomial - $(x+4)(x+2)^2(x-3)$

actual y-intercept = $(4)(2)^2(-3) = (4)(4)(-3) = -48$

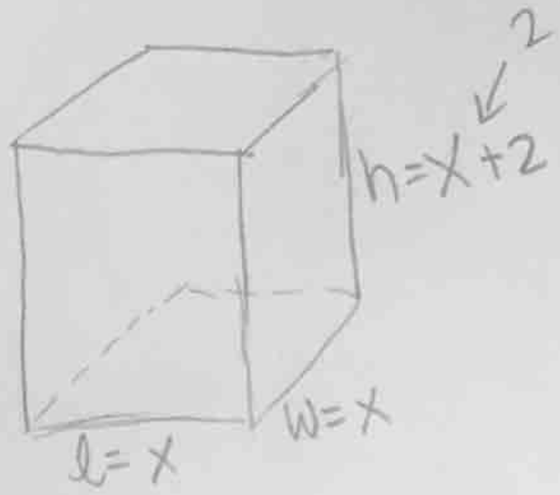
multiply constants together

wanted y-intercept based on graph $y = -4$

$$\frac{1}{12}(x+4)(x+2)^2(x-3)$$

$$\frac{\text{wanted } -4}{\text{actual } -48} = \frac{1}{12}$$

13)



2 inches more

a) $V = l \cdot w \cdot h$
$= (x)(x)(x+2)$

b) $x = 5 \rightarrow (5)(5)(5+2) = (5)(5)(7) = 175 \text{ in}^3$

c) $45 = (x)(x)(x+2)$
$45 = (x^2)(x+2)$
$45 = x^3 + 2x^2$
$-45 \quad -45$

$0 = x^3 + 2x^2 - 45$ graph!

$x = 3$

$l = 3$ in