Rational functions Practice Test

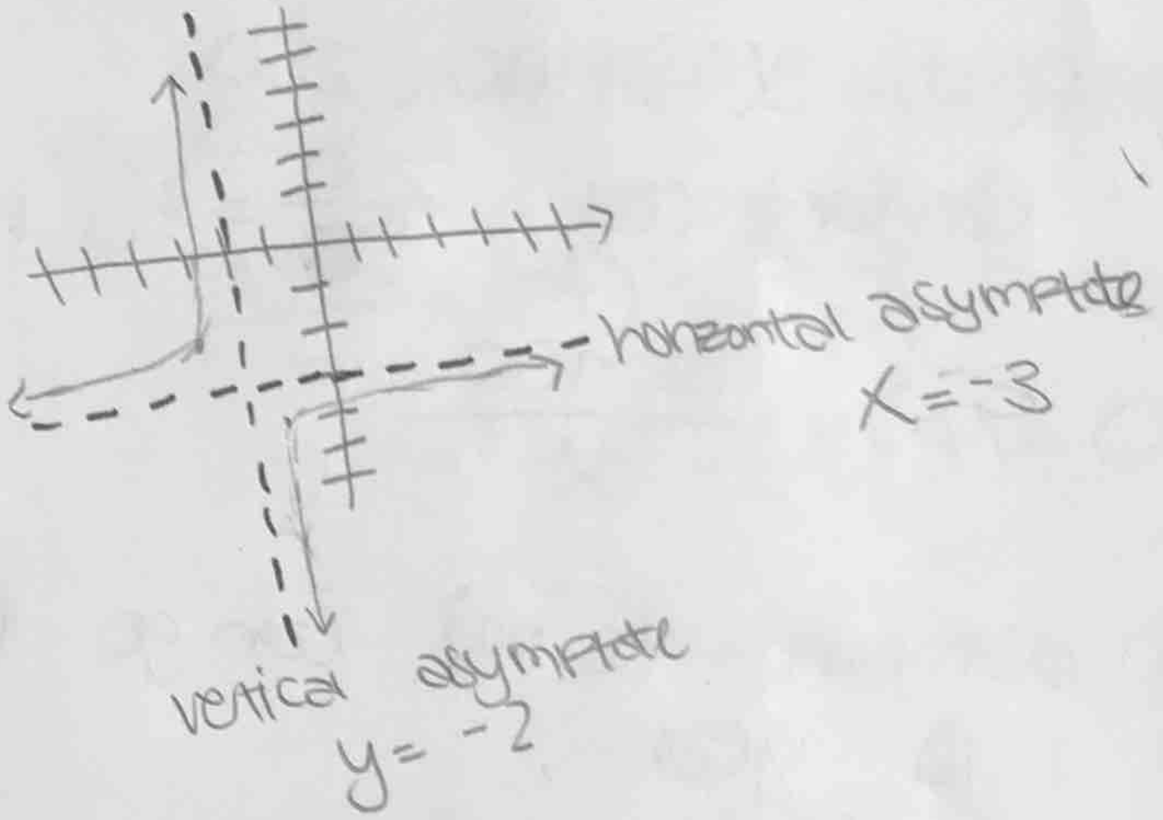
1) $\frac{m^4}{4n^3} \cdot \left(\frac{2n}{m^3}\right)^3 = \frac{m^4}{4n^3} \cdot \frac{(2)^3(n)^3}{(m^3)^3} = \frac{m^4}{4n^3} \cdot \frac{8n^3}{m^9} = \frac{8n^3m^4}{4n^3m^9}$

$= \frac{8}{4} \cdot \frac{n^3}{n^3} \cdot \frac{m^4}{m^9} = 2 \cdot (1) \cdot m^{-5} = 2 \cdot \frac{1}{m^5} = \boxed{\frac{2}{m^5}}$

2) $\frac{(p^2q)^{-1}}{p^2q^{-1}} = \frac{(p^2)^{-1}(q)^{-1}}{p^2q^{-1}} = \frac{p^{-2}q^{-1}}{p^2q^{-1}} = \frac{p^{-2}}{p^2} \cdot \frac{q^{-1}}{q^{-1}} = p^{-4}(1) = \boxed{\frac{1}{p^4}}$

3) $f(x) = -\frac{1}{x+2} - 3$

flipped ↑ left 2 ↑ down 3



4) domain - (x-values) -

$(-\infty, -2), (-2, \infty)$

range - (y-values) -

$(-\infty, -3), (-3, \infty)$

x-Intercept (y=0) -

$0 = -\frac{1}{x+2} - 3$

$+3 \quad +3$

$3 = -\frac{1}{x+2}$

$\cdot(x+2) \quad \cdot(x+2)$

$3(x+2) = -1$

$3x + 6 = -1$

$-6 \quad -6$

$\frac{3x}{3} = \frac{-7}{3}$

$x = -\frac{7}{3}$

y-intercept (x=0) -

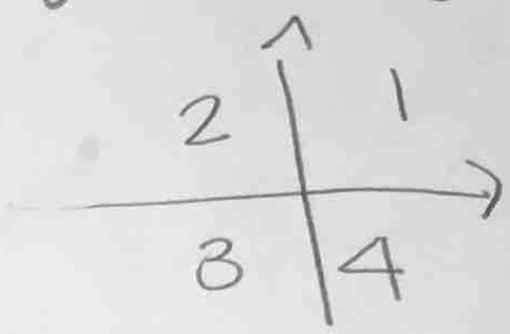
$f(x) = -\frac{1}{(0)+2} - 3$

$= -\frac{1}{2} - 3$

$= -\frac{7}{2}$ or -3.5

5) No x-intercepts, only in first and second quadrants

expected graph:



$f(x) = \frac{1}{x^2}$

6) $f(x) = 3x + 100$ $f(x)$ = total cost, x = # of classes

a) $f(10) = 3(10) + 100 = 130$

average cost $= \frac{130}{10} = \$13$ Per class

b) $f(50) = 3(50) + 100 = 250$

average cost $= \frac{250}{50} = \$5$ Per class

c) $C(x) = \frac{3x + 100}{x}$

d) domain - $[0, \infty)$, can go to 0 to ∞ classes within a year.

e) $f(x) = 6x$

$6x < 3x + 100$

$-3x \quad -3x$

$\frac{3x}{3} < \frac{100}{3}$

$x < 33.\overline{3}$

When you go to less classes than 33, it is a better deal.

7) $\frac{y^2 - 4}{y^2 + y - 6} = \frac{(y+2)(y-2)}{(y+3)(y-2)} = \frac{y+2}{y+3}$

8) $\frac{x+1}{6} = \frac{x}{1} - \frac{3x-2}{4}$ ← common denominator = 12

$\frac{2(x+1)}{2(6)} = \frac{12(x)}{12(1)} - \frac{3(3x-2)}{3(4)}$ ← multiply numerator and denominator by same # to get common denominator

$\frac{2x+2}{12} = \frac{12x}{12} - \frac{9x-6}{12}$

$2x+2 = 12x - (9x-6)$ ← ignore denominator and solve for x.

$2x+2 = 3x+6$
$-2x \quad -2x$

$2 = x+6$
$-6 \quad -6$

$-4 = x$

Check: $x = -4$

$\frac{(-4)+1}{6} = (-4) - \frac{3(-4)-2}{4}$

$-\frac{1}{2} = -4 - (-3.5)$

$-\frac{1}{2} = -\frac{1}{2}$ ✓

9) $\frac{2}{x+2} + \frac{x^2}{x^2-4} = \frac{1}{x-2}$ ← $x^2-4 = (x+2)(x-2)$

$\frac{2}{x+2} + \frac{x^2}{(x+2)(x-2)} = \frac{1}{x-2}$ ← common denominator = $(x+2)(x-2)$

$\frac{2(x-2)}{(x+2)(x-2)} + \frac{x^2}{(x+2)(x-2)} = \frac{1(x+2)}{(x-2)(x+2)}$ ← multiply numerator and denominator by same # to get common denominator

$\frac{2x-4}{(x+2)(x-2)} + \frac{x2}{(x+2)(x-2)} = \frac{x+2}{(x-2)(x+2)}$

$2x-4+x^2 = x+2$ ← ignore denominator and solve for x!
$-x \quad\quad -x$

$x-4+x^2 = 2$
$-2 \quad -2$

$x^2+x-6 = 0$

$x^2+x-6=0$ ← add to make b=1, multiply to make c=-6

$3+-2=1$ $\quad 3\cdot -2=-6$

$(x+3)(x-2)=0$

$x+3=0$ $\quad x-2=0$

$-3\ -3$ $\quad +2\ +2$

$x=-3$ $\quad x=2$

Check: $\boxed{x=-3}$

$$\frac{2}{(-3)+2}+\frac{(-3)^2}{(-3)^2-4}=\frac{1}{(-3)-2}$$

$$\frac{2}{-1}+\frac{9}{9-4}=\frac{1}{-5}$$

$$-2+\frac{9}{5}=-\frac{1}{5}$$ ✓

$$-\frac{1}{5}=-\frac{1}{5}$$

~~$x=2$~~

$$\frac{2}{2+2}+\frac{2^2}{2^2-4}=\frac{1}{2-2}$$

$$\frac{2}{4}+\frac{4}{0}=\frac{1}{0}$$

undefined!

extraneous solution!