Linear systems of equations test review notes

***WHAT IS A SYSTEM OF EQUATIONS?

They are multiple equations that share a common solution

EXAMPLES

1) Is $\overset{(x,y)}{(3,6)}$ a solution to this system of equations?

$$\begin{cases} 2x + 7y = 48 \\ 5x + 3y = 51 \end{cases}$$

$\rightarrow$ $2(3) + 7(6) = 48$

$6 + 42 = 48$ True statement!

$48 = 48$ ✓

$\downarrow$

$5(3) + 3(6) = 51$

$15 + 18 = 51$ NOT a true statement!

$33 \neq 51$ X

*(3,6) is not a solution because it does not satisfy both equations. (NOT both true)

2) Is $\overset{(x,y)}{(-2,7)}$ a solution to this system of equations?

$$\begin{cases} -x + y = 9 \\ y = -3x + 1 \end{cases}$$

$\rightarrow$ $-(-2) + (7) = 9$ true statement!

$2 + 7 = 9$

$9 = 9$ ✓

$\downarrow$

$7 = -3(-2) + 1$ true statement!

$7 = 6 + 1$

$7 = 7$ ✓

*(-2,7) is a solution because it satisfies both equations (both are true!)

Methods for solving systems of equations?

1) Graphing
2) Substitution
3) Elimination

* Graphing linear systems of equations -

$$y = mx + b$$

- slope
- "per"
- $\frac{rise}{run}$

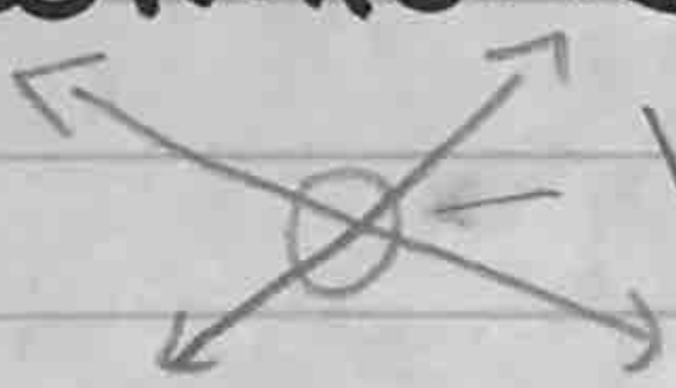
- y-intercept
- start value

* Steps to solve by graphing:

1) make sure both equations are in $y=mx+b$ form
2) graph both lines on the same coordinate plane
   - starting at y intercept $(0,b)$
   - moving with slope $\frac{rise}{run}$
3) find out where they intersect, that is the solution to the system

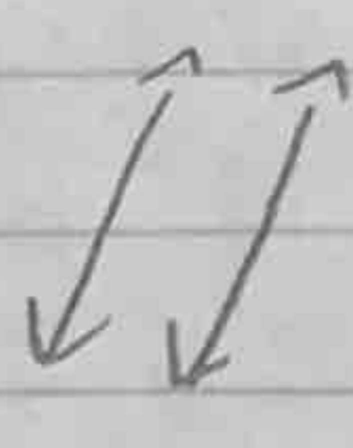
* Number of solutions:

- One -



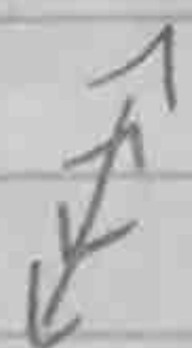
intersects at one point

- None -



- will never intersect (parallel lines)

- Infinite -



- same exact line! (intersects everywhere)

EXAMPLES

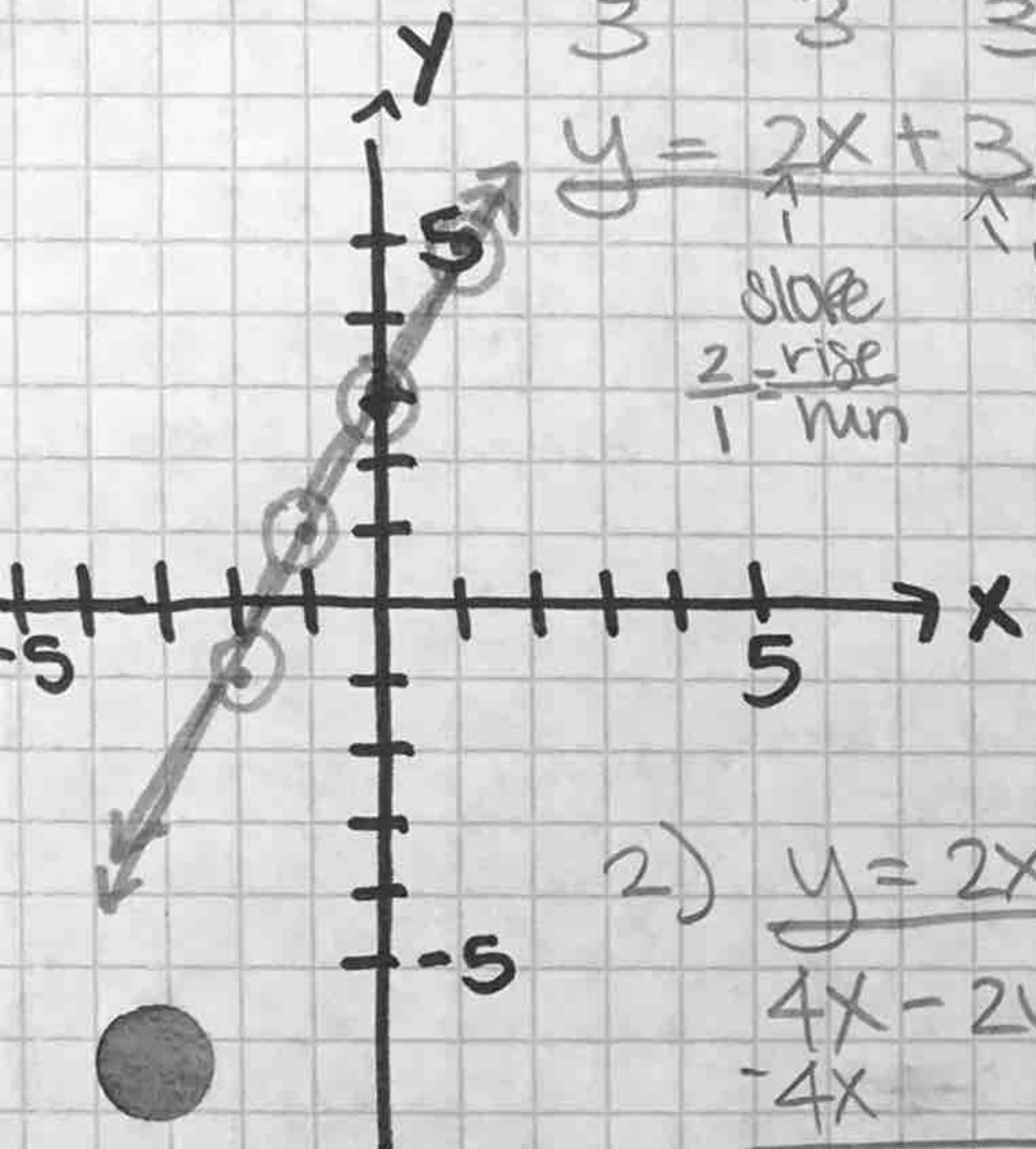
1) $y = 2x + 3$ <- slope intercept form ✓

$3y - 6x = 9$ <- Not slope intercept form. solve for y!

$\quad +6x \quad +6x$

1) move x value by +/-
2) divide by coefficient of y (# in front of y)

$\frac{3y}{3} = \frac{6x}{3} + \frac{9}{3}$



$y = 2x + 3$ <- slope intercept form ✓

slope $\frac{2}{1} = \frac{\text{rise}}{\text{run}}$

y-int (0,3)

** Same exact line, ∞ (infinite) solutions

2) $y = 2x + 3$ ✓

$4x - 2y = 8$

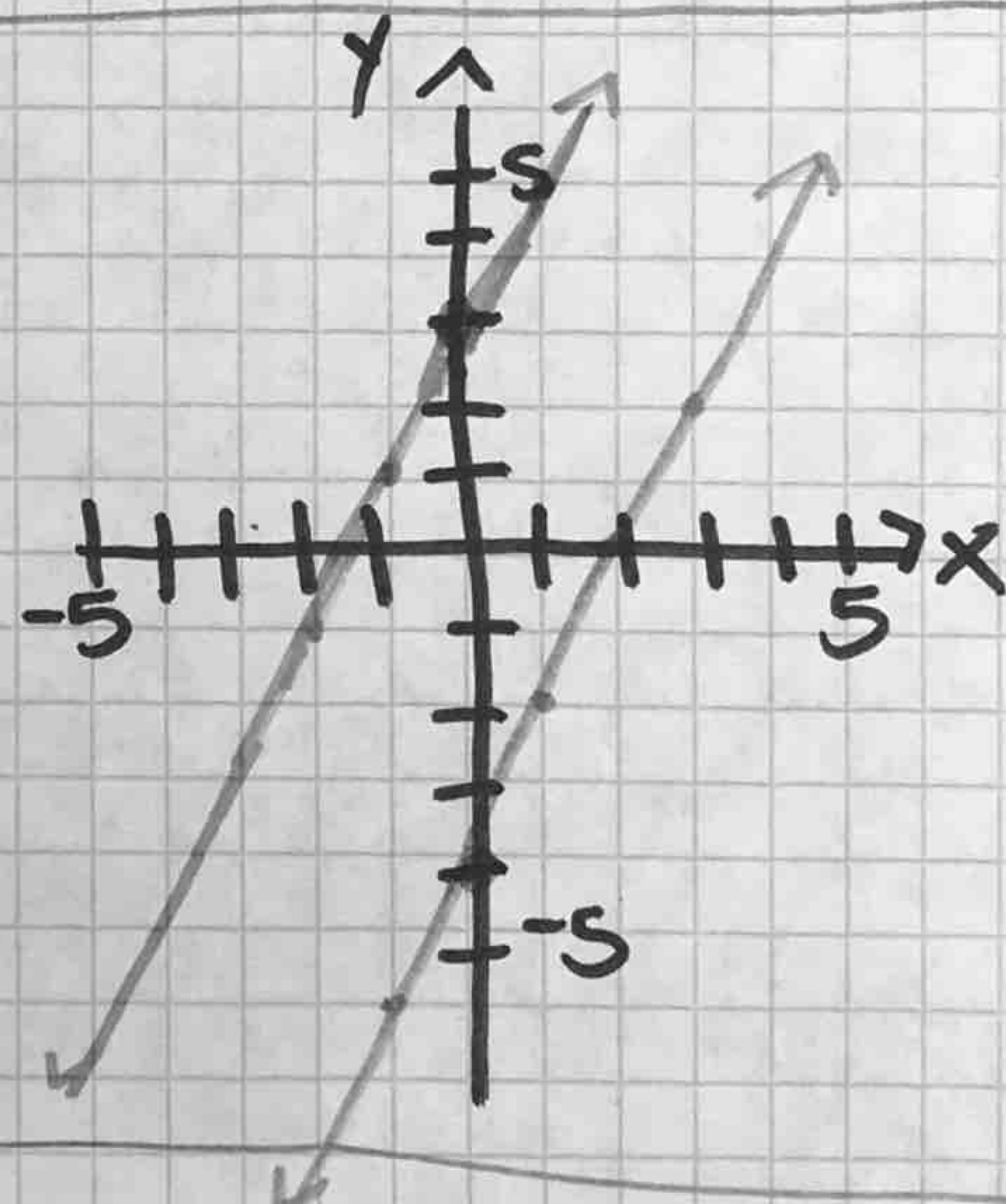
$-4x \quad -4x$

$\frac{-2y}{-2} = \frac{-4x}{-2} + \frac{8}{-2}$

$y = 2x - 4$

slope $\frac{2}{1}$

y-int (0,-4)

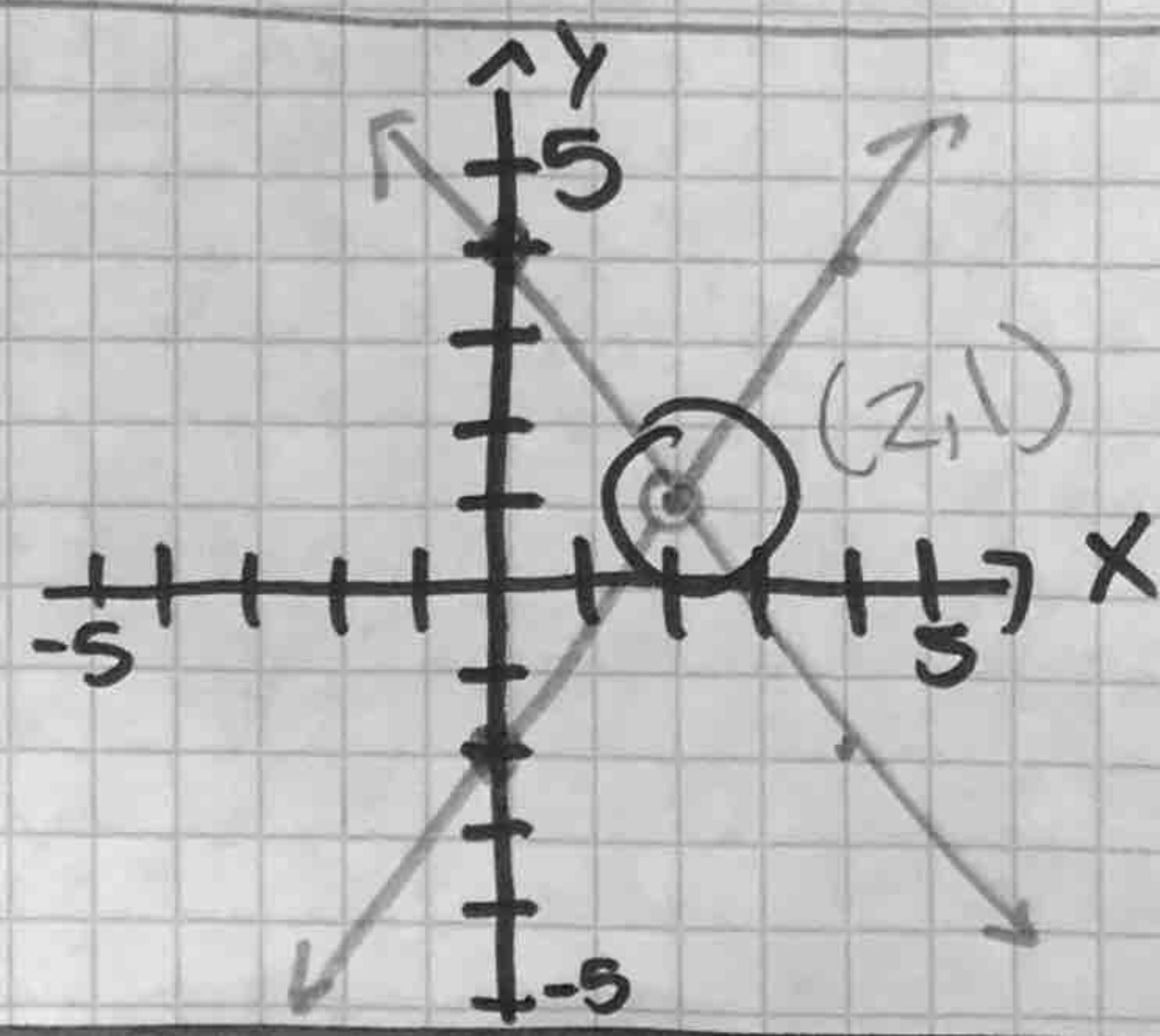


** NO Solution, they are parellel and will never intersect

3) $y = -3/2x + 4$

$y = 3/2x - 2$

*one solution at (2,1)



* solving linear systems of equations using substitution

Take one equation and replace the variable in the other equation with itself.

* steps to solve by substitution:

1) make sure one of your variables is isolated ($x=$ or $y=$)
2) substitute in x or y into the other equation in the system
3) solve multistep equation for variable
   - distribute if you see ( )
   - combine like terms
   - add / subtract
   - multiply / divide
4) once solved for x or y, plug your solution into one of the equations from #1 to solve for the other
5) rewrite solution as a coordinate pair (x, y)

EXAMPLES

1) $y = -3x + 5$

$5x - 4y = -3$

$5x - 4(-3x + 5) = -3$ <- distribute

$5x + 12x - 20 = -3$ <— combine like terms

$17x - 20 = -3$ <— undo +/−

$\quad + 20 \quad +20$

$$\frac{17x}{17} = \frac{17}{17}$$ ← undo ×/÷

$x = 1$

Plug back into one of the original equations

$y = {}^-3x + 5$
$y = {}^-3(1) + 5$
$y = {}^-3 + 5$
$y = 2$

$\boxed{\text{Solution} - (x, y) \quad (1, 2)}$

2) $-5x + y = -3$
$3x - 8y = 24$

* Must isolate a variable, so look for a variable with a coefficient of 1 (no other # in front of it)

$-5x + y = -3$
$+5x \qquad +5x$

$y = 5x - 3$

$3x - 8y = 24$
$3x - 8(5x - 3) = 24$ ← distribute
$3x - 40x + 24 = 24$ ← combine like terms
$-37x + 24 = 24$ ← undo +/−
$\quad - 24 \quad -24$

$$\frac{-37x}{-37} = \frac{0}{-37}$$ ← undo ×/÷

$x = 0$

Plug back into one of the original equations

$y = 5x - 3$
$y = 5(0) - 3$
$y = {}^-3$

$\boxed{\text{solution} - (x, y) \quad (0, {}^-3)}$

* Solving linear systems of equations using elimination

Eliminate a variable by adding or subtracting vertically. The coefficients (#'s in front of variable) MUST form a zero pair.

* Steps to solve by elimination:
  1) look at the coefficients in front of x and y. make sure the coefficients form a zero pair when adding or subtracting (may need to multiply one if not both to make a zero pair)

EXAMPLES: *highlighted the zero pairs

no multiplying

$$4x + 2y = 6$$
$$-4x - 8y = 12$$

$$3x - 8y = 2$$
$$4x + 8y = 5$$

multiply once

$$2(2x + 3y = 12) \Rightarrow 4x + 6y = 24$$
$$4x + 4y = 8 \Rightarrow 4x + 4y = 8$$

2 goes into 4! 2 times, so must multiply top equation by 2

3 does not go into 4 and 4 does not go into 3

multiply both

$$3x + 2y = 10$$
$$4x + 3y = 11$$

eliminate x

$$4(3x + 2y = 10)$$
$$3(4x + 3y = 11)$$

eliminate y

$$3(3x + 2y = 10)$$
$$2(4x + 3y = 11)$$

* multiply by other coefficient!!

$12x + 8y = 40$
$12x + 9y = 33$

$9x + 6y = 30$
$8x + 6y = 22$

2) Once you have formed a zero pair in front of your x's or your y's, add or subtract vertically to eliminate the variable.

** Remember: (-, - or +, +)
- If they are the same sign, subtract vertically.
- If they are different signs (+, -) add vertically.

3) Once solved for x or y, plug your solution into one of the equations to solve for the other

4) Rewrite solution as a coordinate pair (x, y)

EXAMPLES

1) $3(-7x + y = -19)$ → $-21x + 3y = -57$
$-2x + 3y = -19$ → $-\ \ -2x + 3y = -19$

must subtract because they're the same sign

$\frac{-19x}{-19} = \frac{-38}{-19}$

$x = 2$

plug back into one of the original equations

NO zero pairs, must multiply! can multiply top equation by 3 to eliminate the other 3y!

$-2x + 3y = -19$
$-2(2) + 3y = -19$
$-4 + 3y = -19$
$+4 \qquad\quad +4$

$\frac{3y}{3} = \frac{-15}{3}$

$y = -5$

Solution - (x, y)
(2, -5)

*Systems of equations: real world applications

*Steps to solve word problems:

1) Write 2 equations to represent the 2 different situations. remember the options:

$$\underbrace{Ax + By}_{\text{what you're adding together}} = \underset{\text{total}}{C}$$

$$y = \underset{\substack{\text{slope} \\ \text{"per"}}}{m}x + \underset{\text{start value}}{b}$$

2) Solve using any of the 3 methods: graphing, elimination, or substitution.

EXAMPLES

1) Budget Bumpers

$5 gas fee,
$0.50 each additional mile

$$\underset{\text{cost}}{y} = \underset{\text{"per"}}{0.50}\underset{\text{miles}}{x} + \underset{\text{starting cost}}{5}$$

Reliable rides

$1.00 per mile,
no flat rate

$$\text{cost} \rightarrow y = \underset{\text{"per"}}{1}\underset{\text{miles}}{x} + \underset{\text{starting cost}}{0}$$

When will they be equal?

$$y = y$$

$$0.50x + 5 = 1x$$

$$\begin{array}{r|l} -5 & -5 \\ \hline 0.50x = & 1x - 5 \\ -1x & -1x \\ \hline \frac{-0.50x}{-0.50} = & \frac{-5}{-0.50} \\ x = & 10 \end{array}$$

plug back into an original equation

$y = 1x$
$y = 1(10)$
$y = 10$

$$\boxed{\text{Solution - } (10, 10)}$$

10 miles, \$10

2) first night show tickets -
25 children, 33 adults, \$240 total from ticket sales

$$25x + 33y = 240$$

Second night -
10 children, 11 adults, \$85 total from ticket sales

$$10x + 11y = 85$$

Solve, how much are adult tickets (y) and children tickets? (x)

$$25x + 33y = 240 \rightarrow 25x + 33y = 240$$
$$3(10x + 11y = 85) \rightarrow -\ 30x + 33y = 255$$
$$\frac{-5x}{-5} = \frac{-15}{-5}$$
$$x = 3$$

$25(3) + 33y = 240$
$75 + 33y = 240$
$-75 \quad\quad -75$
$\frac{33y}{33} = \frac{165}{33}$
$y = 5$

solution - (3, 5)

children tickets \$3

adult tickets \$5